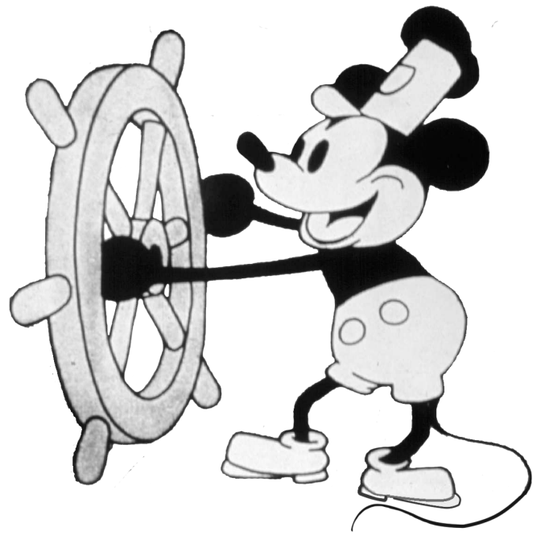

CSCI 461: Computer Graphics

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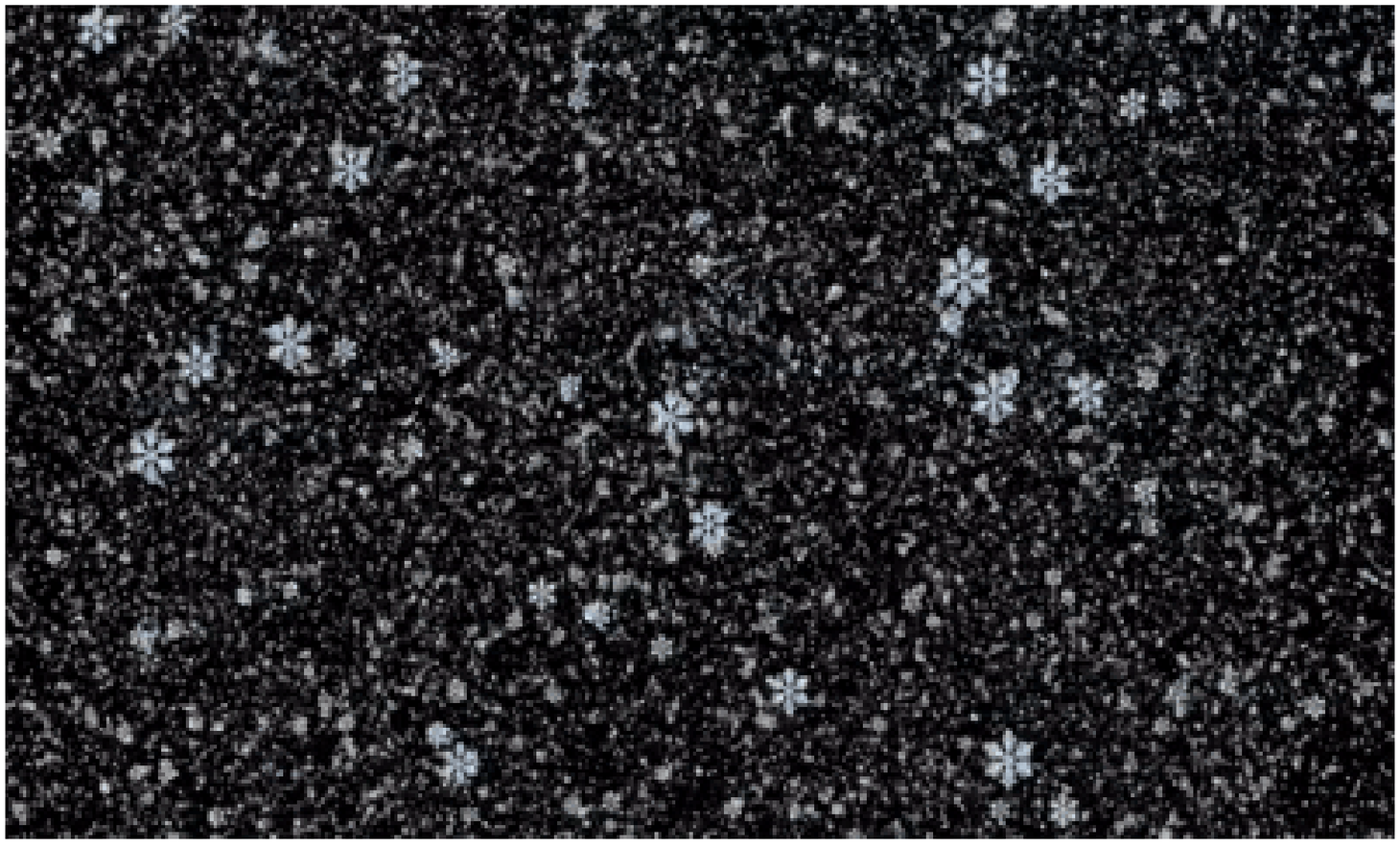
Lecture 12: Animating connected particles

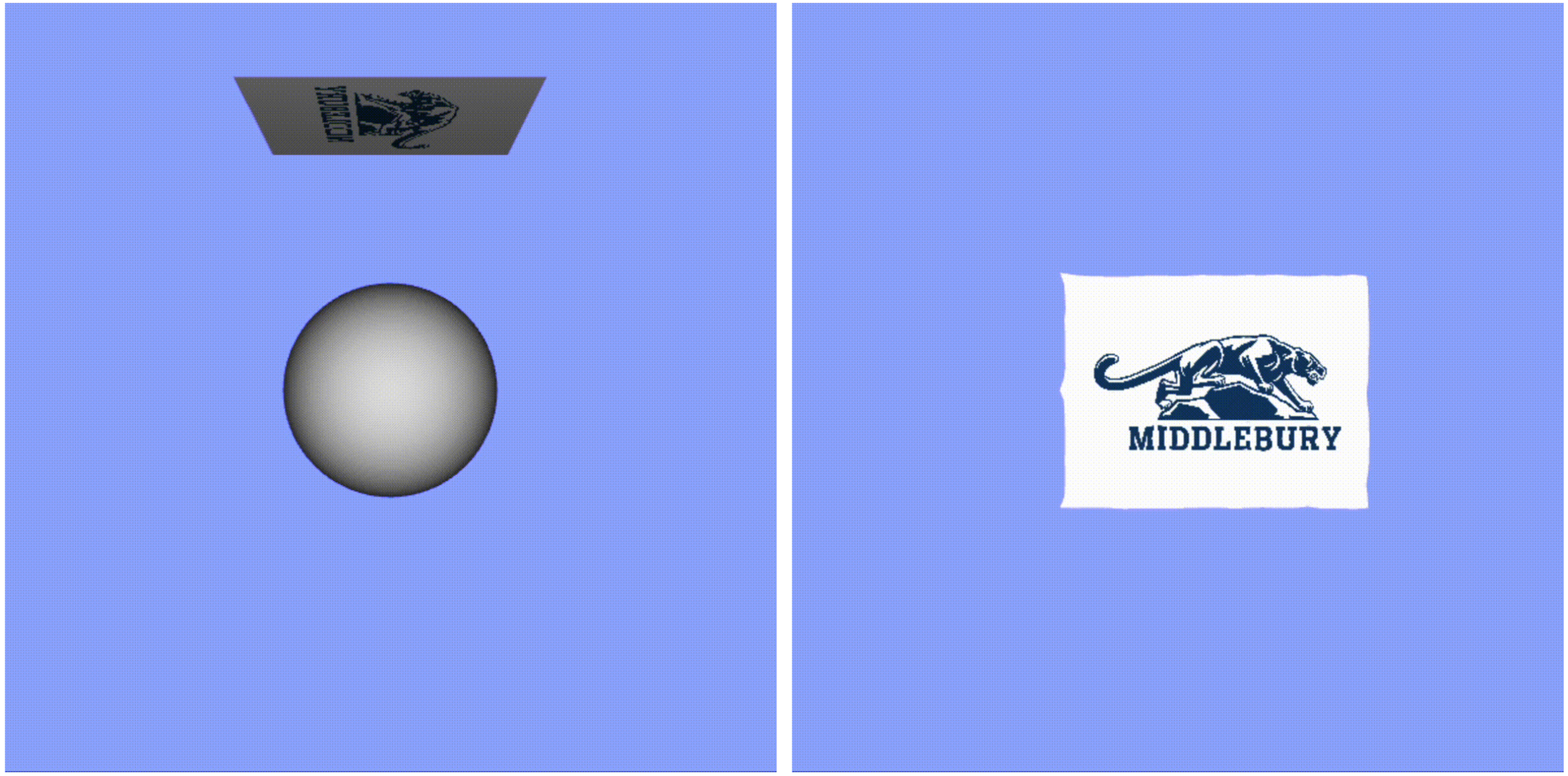
Last time, we computed object motion from external forces (only).



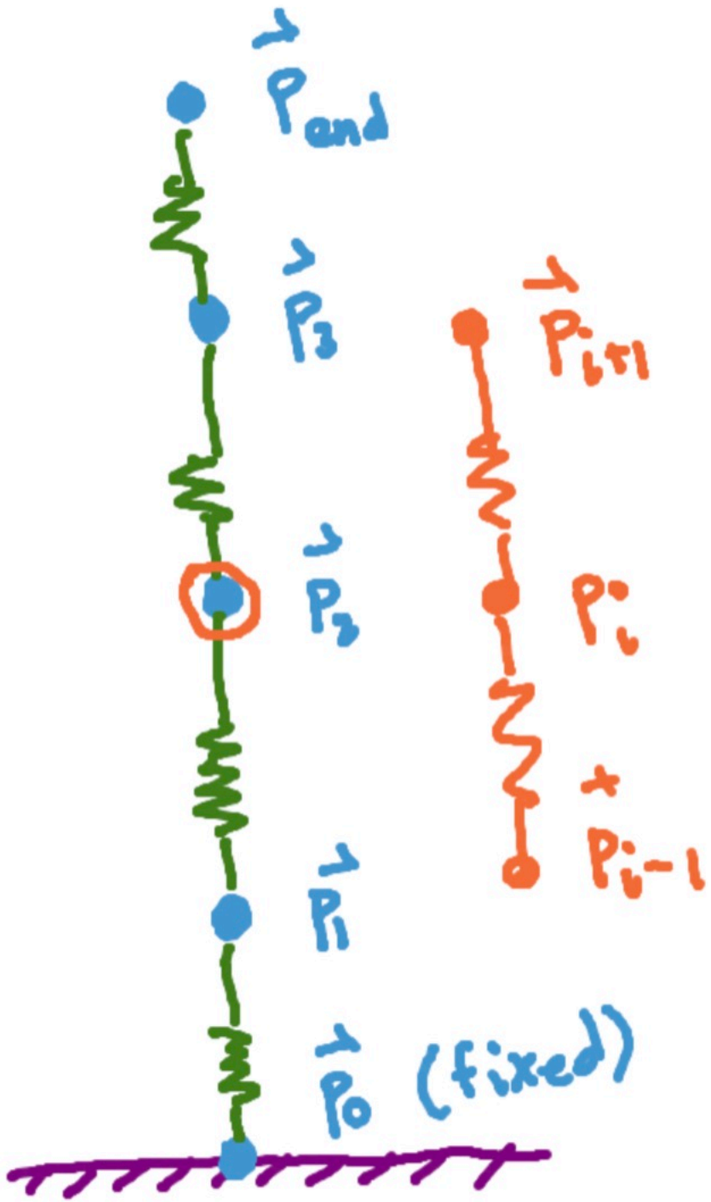
Now we want particles to be **connected** to each other!

By the end of today's lecture, you will be able to:

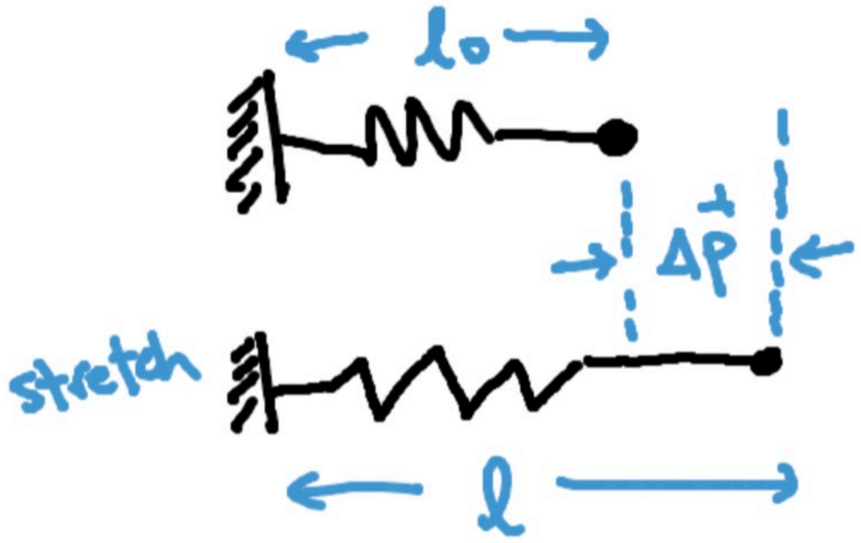
- model **connections** between particles as **springs**,
- use **Verlet integration** to update the position of particles,
- animate **hair** or **cloth**,
- **recap** some of the things we did this semester.



Example: modeling strands of hair with springs.



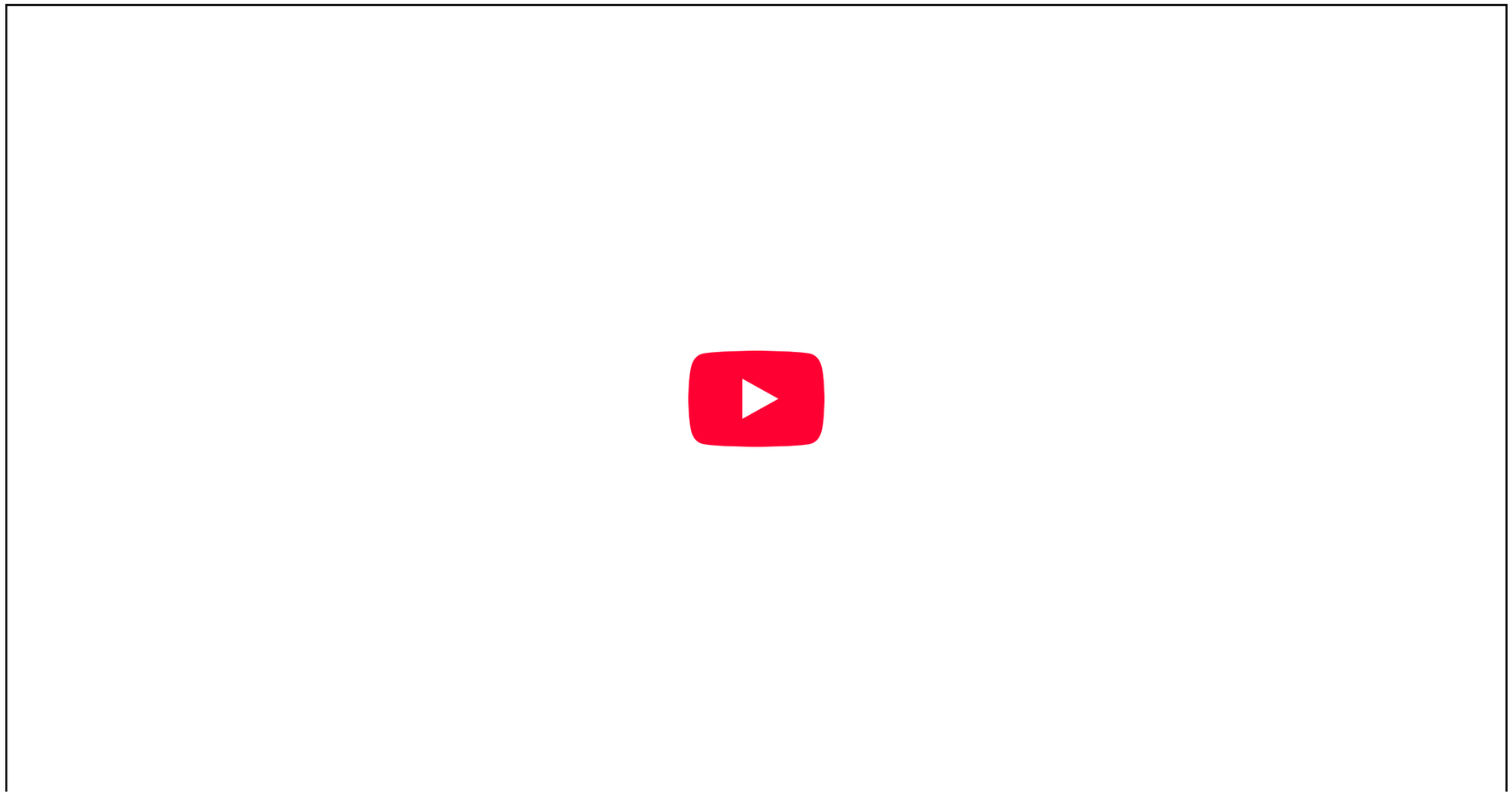
Hooke's law: force is linear with displacement



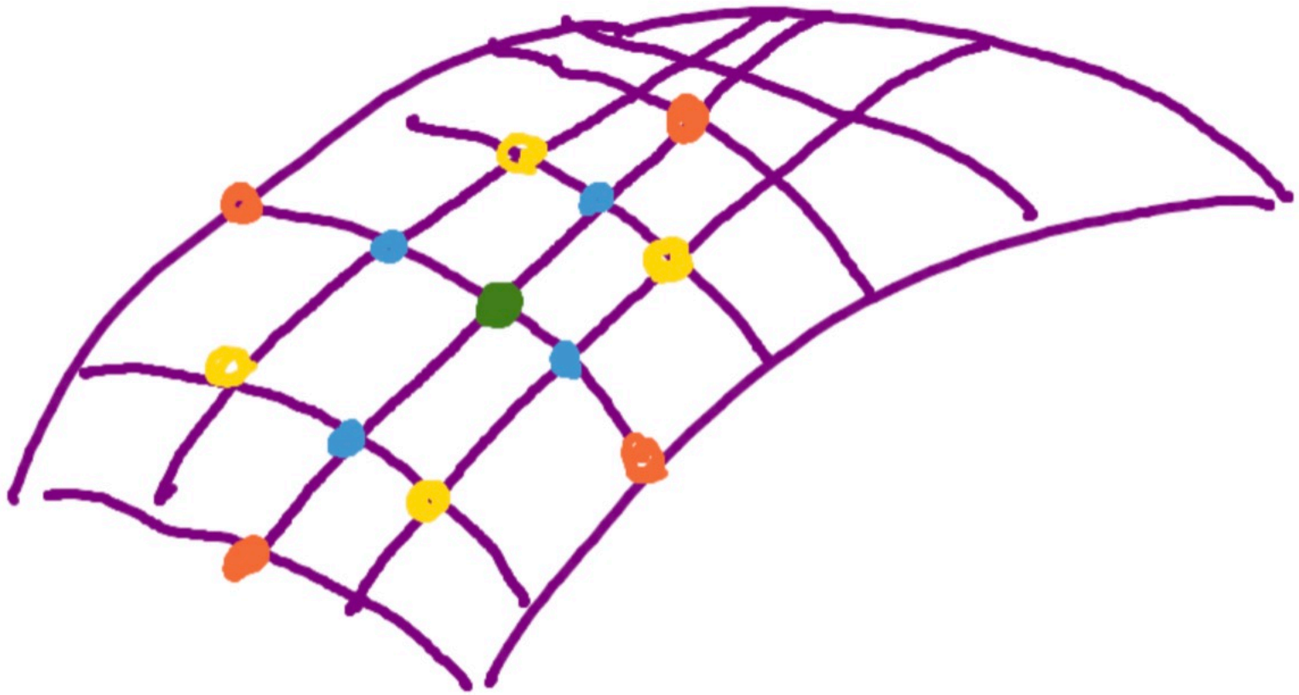
$$\vec{f}_s = K\,\Delta\vec{p}$$

spring constant

Recent research in modeling tightly-coiled hair.



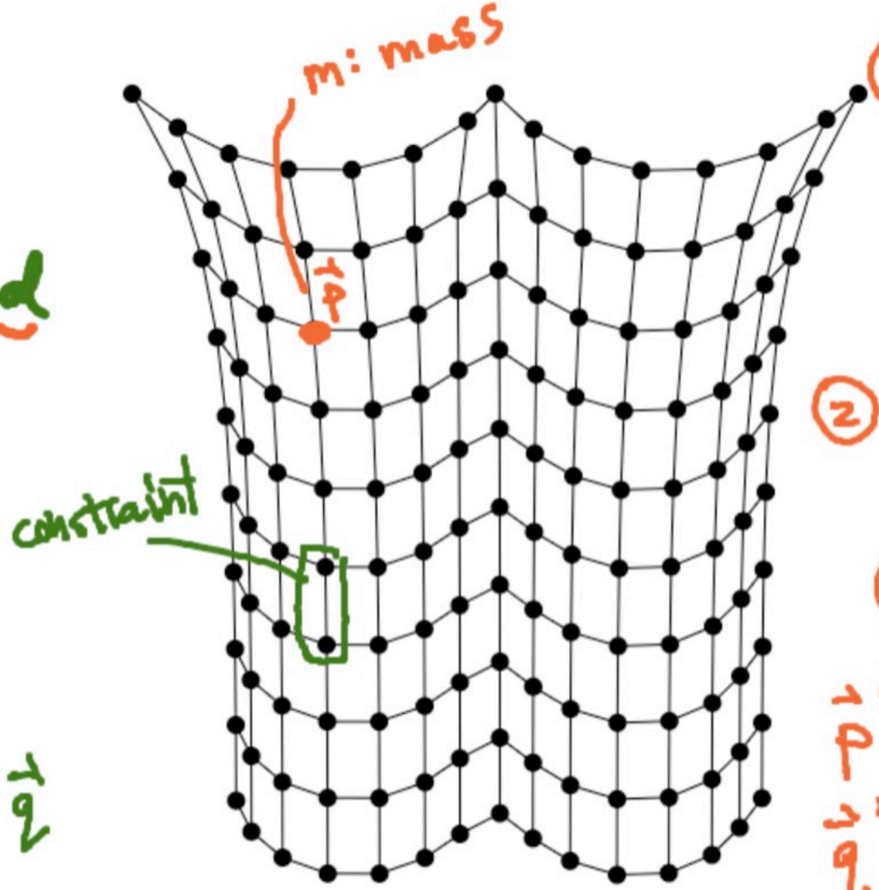
The same idea applies to modeling cloth - but more complicated with bookkeeping for 2d connections.



A simpler way to model cloth: model particles + constraints and step in time with Verlet integration.

2 stages

Stage 1: update particles according to external forces



① $\vec{p}^{\,k+1} = 2\vec{p}^{\,k} - \vec{p}^{\,k-1} + \frac{\sum \vec{f}_{ext}}{m} \Delta t^2$

(verlet integration)

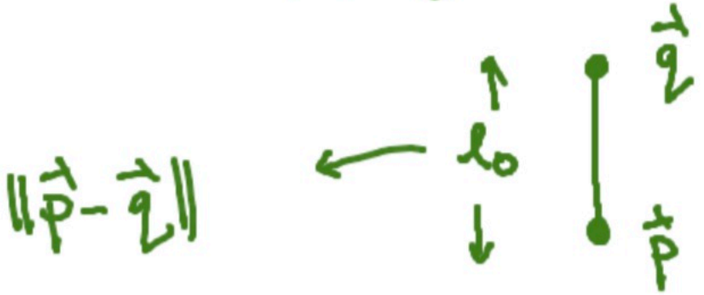
② ⓐ calculate $\delta = \frac{l - l_0}{l_0}$

ⓑ update both $\vec{p}$ and $\vec{q}$

$\vec{p}' = \vec{p} + \alpha_p (\vec{q} - \vec{p})$

$\vec{q}' = \vec{q} - \alpha_q (\vec{q} - \vec{p})$

Stage 2: update constraints (springs)



$\alpha_p = \frac{m_q}{(m_p + m_q)} \delta$ $\alpha_q = \frac{m_p}{(m_p + m_q)} \delta$

$m_p = m_q$?? $\delta/2$

Cloth animation in action!

https://philipclaude.github.io/csci461f25/chapter12/

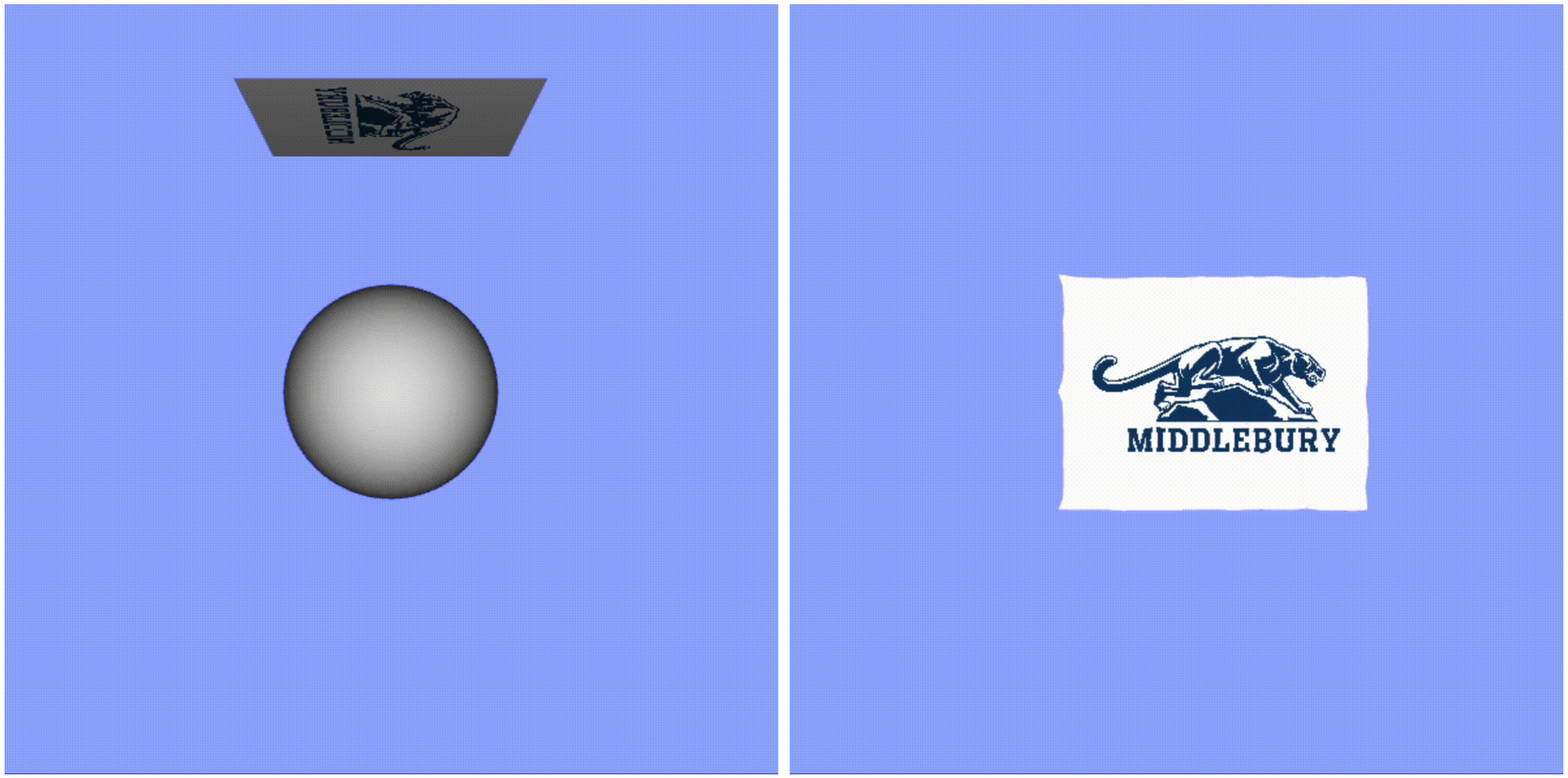
stage 1

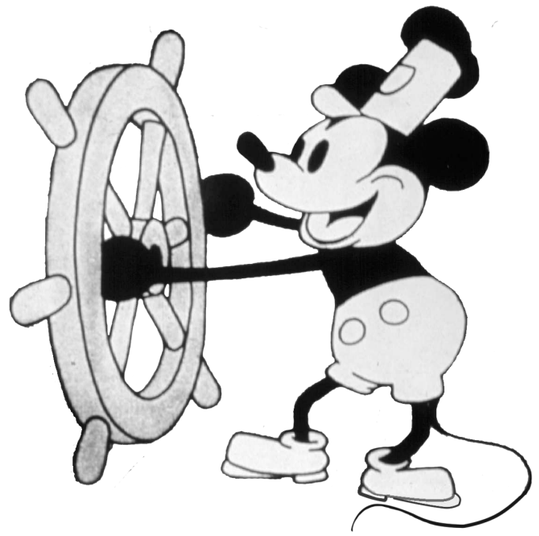
stage 2

```
// cloth update algorithm for one time step (frame)
for numIter iterations: // numIter is usually about 1 or 2
  for each particle:
    particle.forceUpdate();
  for each constraint:
    p = constraint.p; // first endpoint vertex (vec3)
    q = constraint.q; // second endpoint vertex (vec3)
    delta = (||q - p|| - L0)/||q - p||; // vec3
    dx = delta * (q - p); // vec3
    mt = mp * mq / (mp + mq); // mp and mq are the masses of particles p and q
    constraint.p += dx * mt / mp;
    constraint.q -= dx * mt / mq;
```

Summary

- Cloth model involves **points** and **constraints**,
- Update points from **external forces** using **Verlet integration**,
- Update points from **constraint forces**,
- You'll implement these techniques in Thursday's lab.





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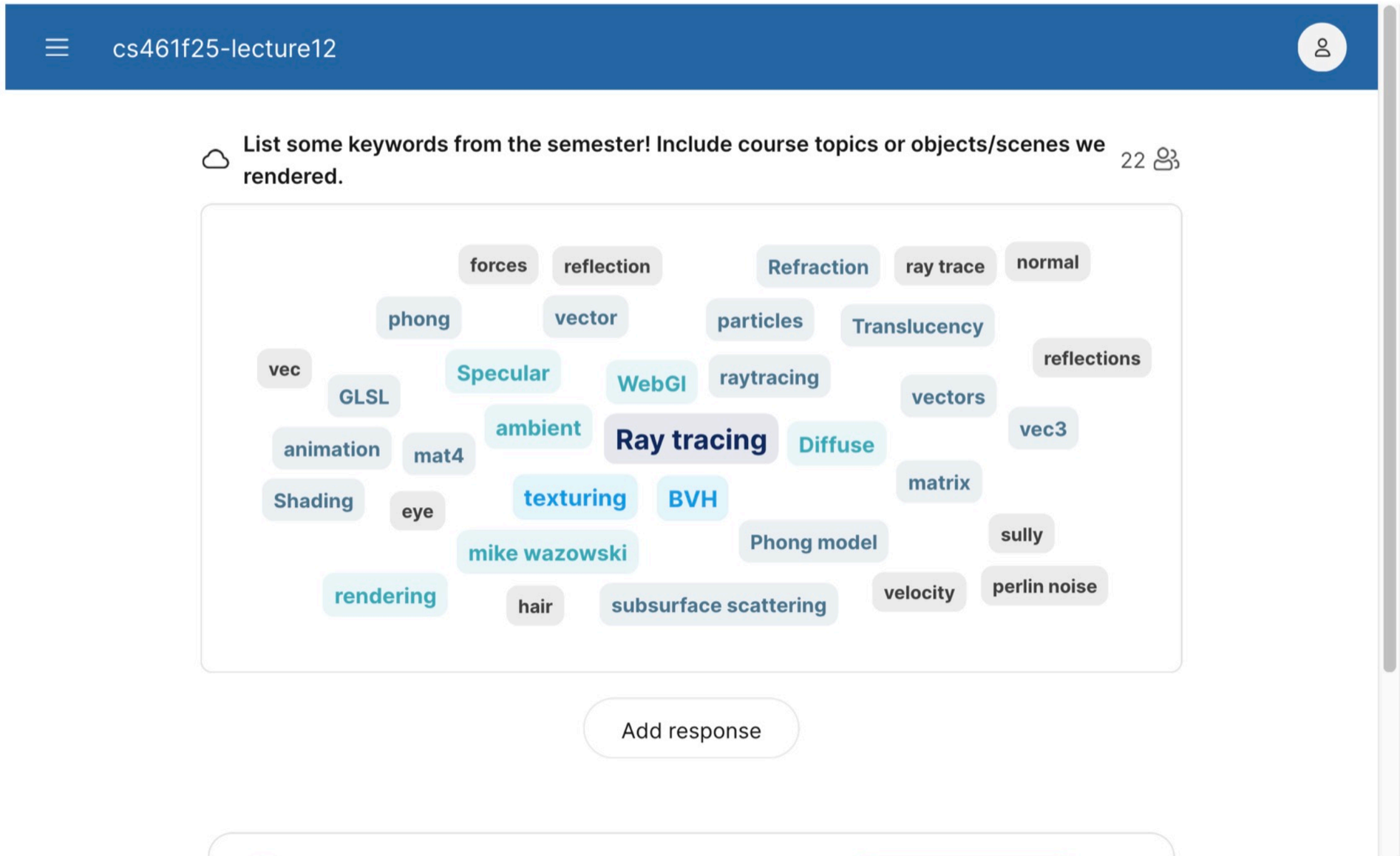
Course Recap

Exercise: list keywords from the course in groups of 2 (slido #3834500).



Where to go from here?

- Try other features from the labs,
- Check out other APIs and tools: `WebGPU`, `OpenGL`, `Shadertoy`, `Three.js`,
- Try implementing labs or Ray Tracing Book series in `C++`.
- Become a student volunteer SIGGRAPH 2026! Applications due in February 2026.

Please complete the Course Response Form (this is the pre-lab this week).

go/crf

(keep screenshot of the confirmation page so you can submit the pre-lab)

Thank you for a fun semester!

Cloth setup for Lab 11.