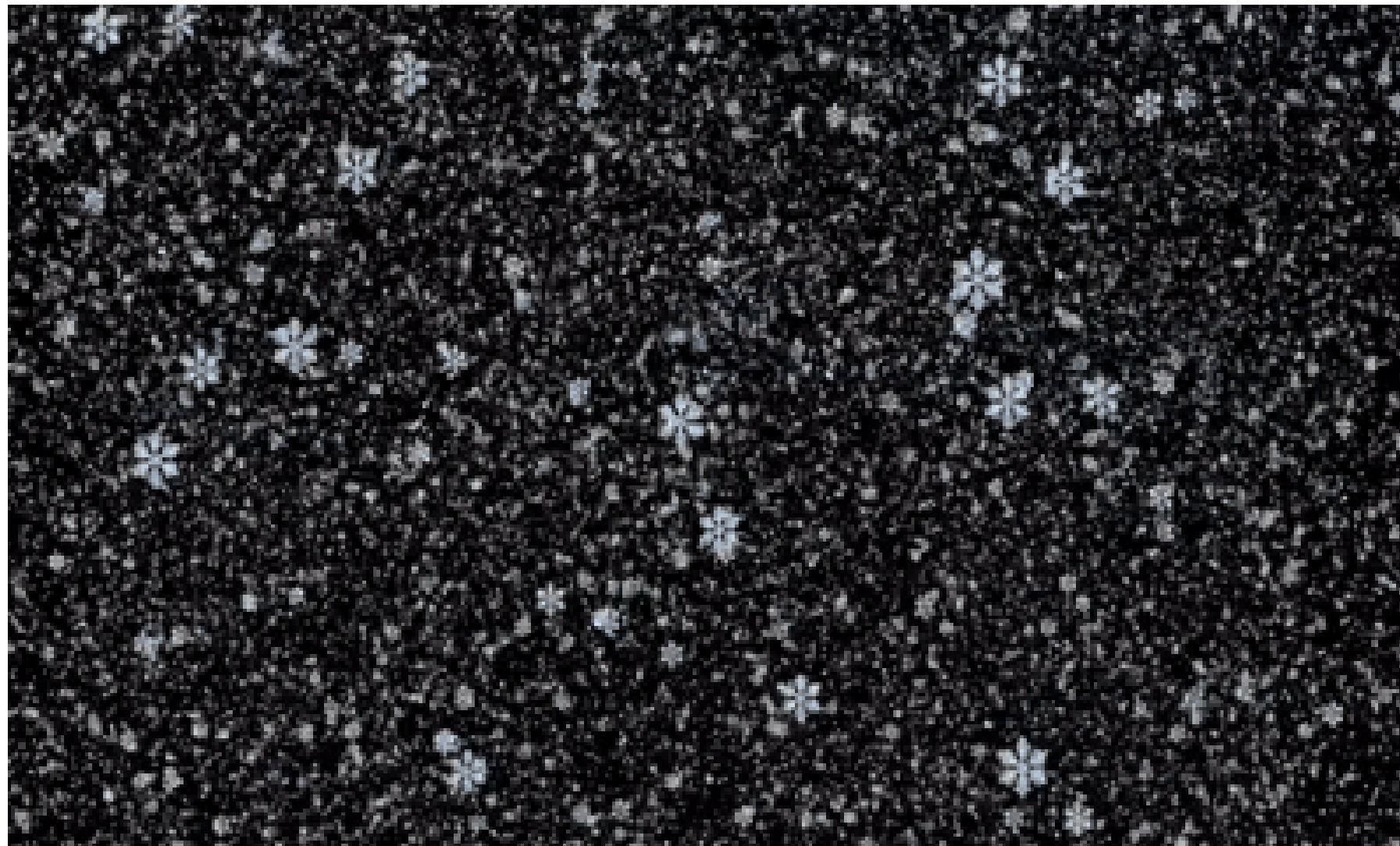


CSCI 461: Computer Graphics

Middlebury College, Fall 2025

Lecture 12: Animating connected particles

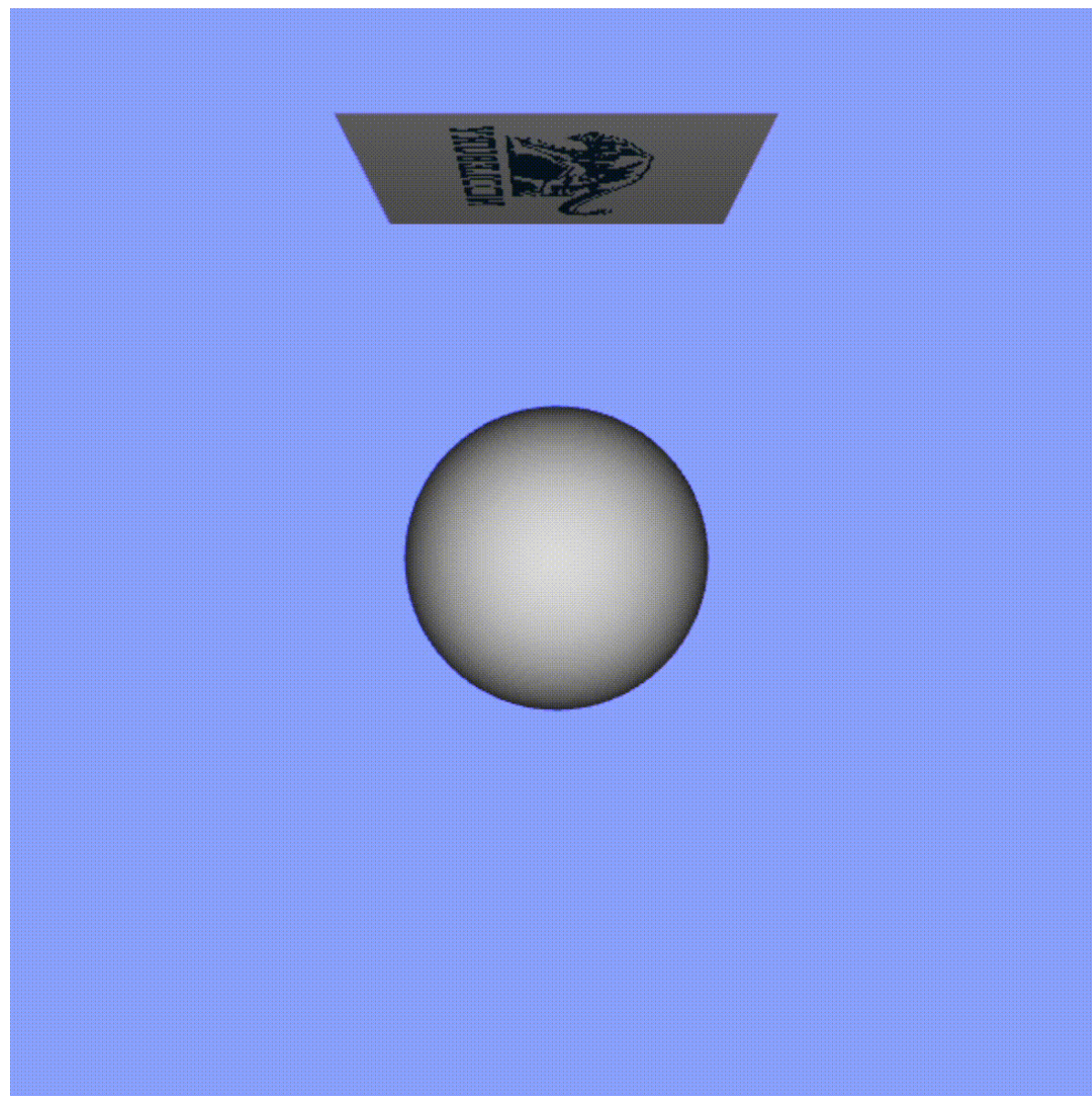
Last time, we computed object motion from external forces (only).



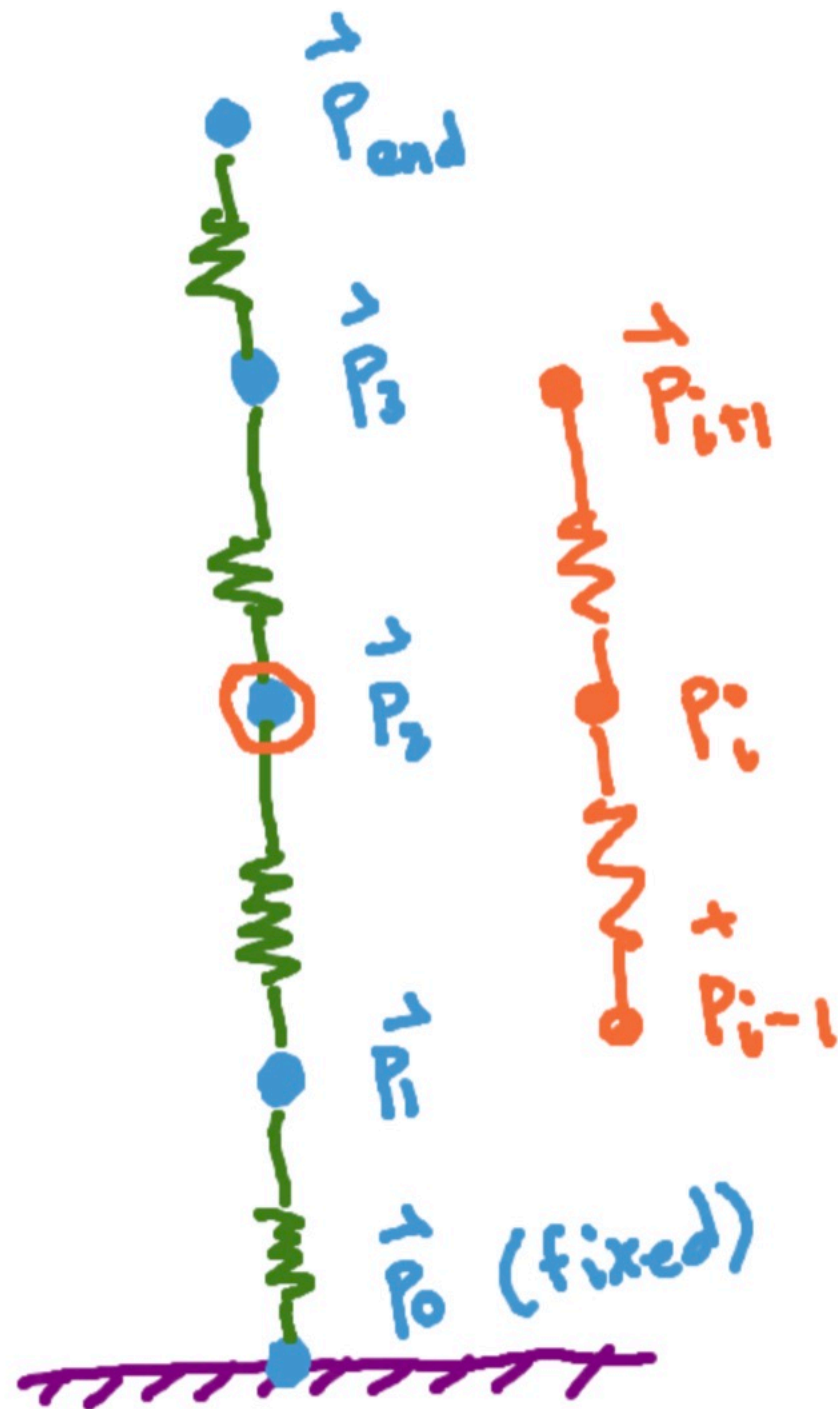
Now we want particles to be **connected** to each other!

By the end of today's lecture, you will be able to:

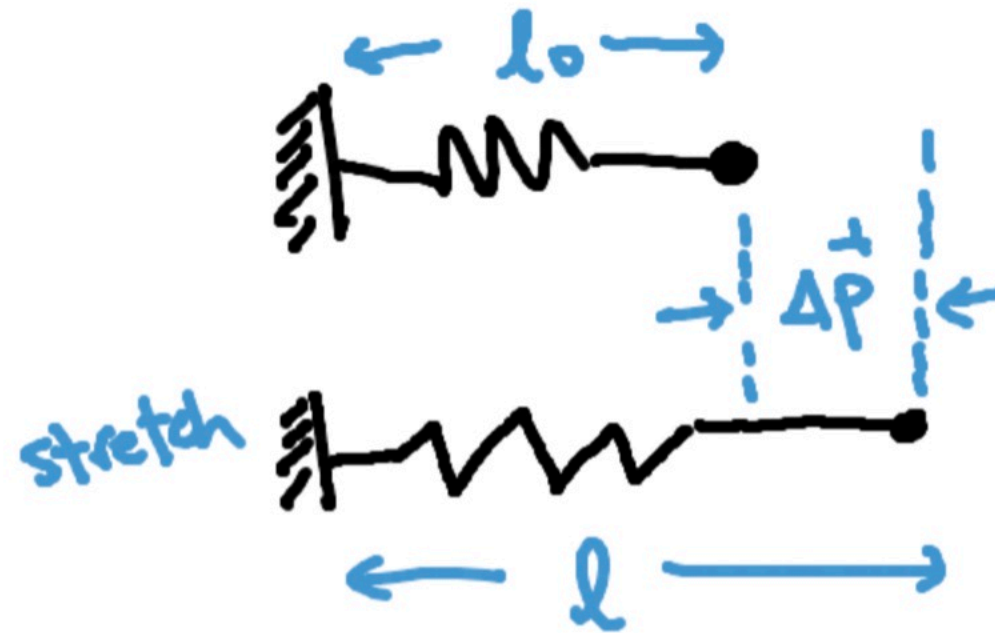
- model **connections** between particles as **springs**,
- use **Verlet integration** to update the position of particles,
- animate **hair** or **cloth**,
- **recap** some of the things we did this semester.



Example: modeling strands of hair with springs.



Hooke's law: force is linear with displacement



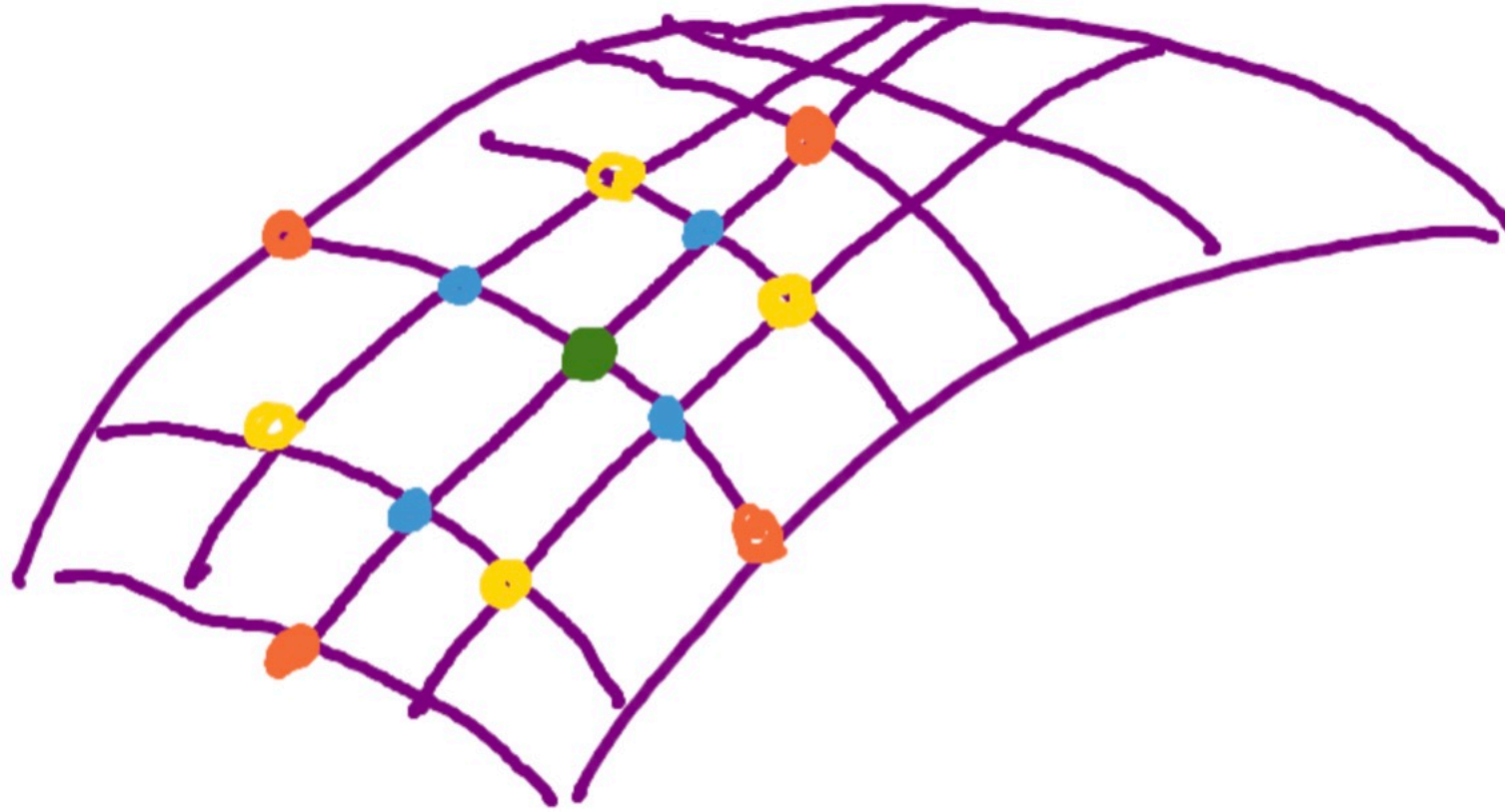
$$\vec{f}_s = K \Delta \vec{p}$$

spring constant

Recent research in modeling tightly-coiled hair.



The same idea applies to modeling cloth - but more complicated with bookkeeping for 2d connections.

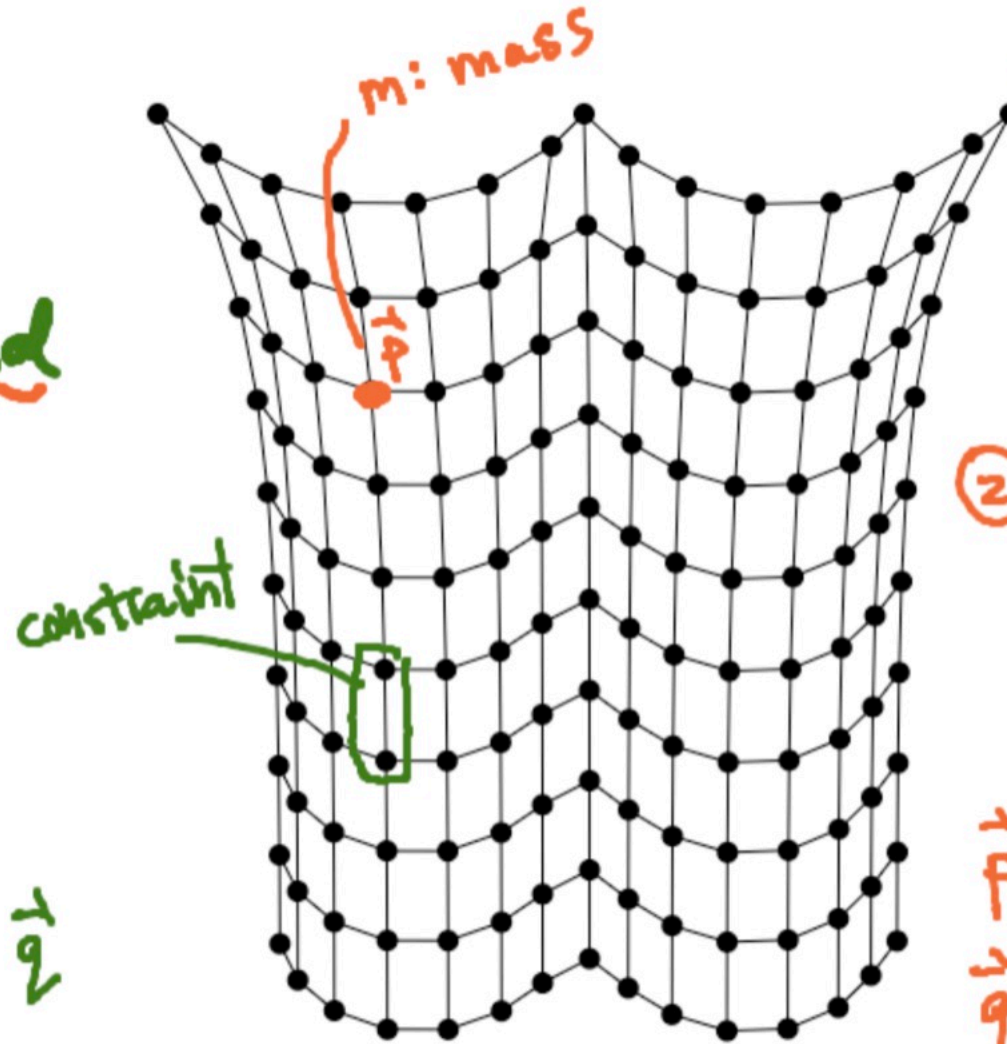


A simpler way to model cloth: model particles + constraints and step in time with Verlet integration.

2 stages

Stage 1: update particles according to external forces

Stage 2: update constraints (springs)



$$\textcircled{1} \vec{p}^{k+1} = 2\vec{p}^k - \vec{p}^{k-1} + \frac{\sum \vec{f}_{\text{ext}} \Delta t^2}{m}$$

(verlet integration)

$$\textcircled{2} \textcircled{a} \text{ calculate } \delta = \frac{l - l_0}{l_0}$$

$$\textcircled{b} \text{ update both } \vec{p} \text{ and } \vec{q}$$

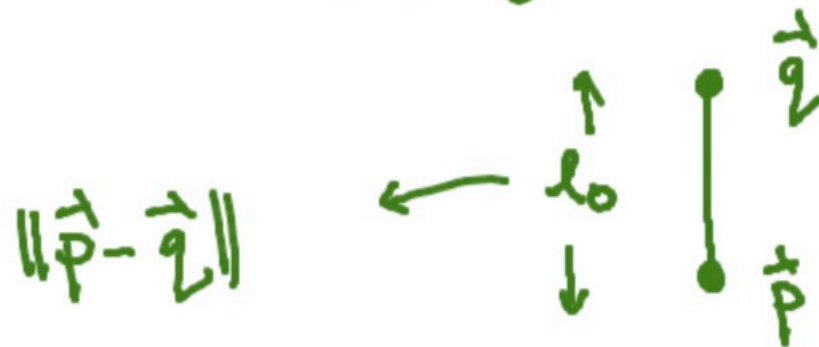
$$\vec{p}' = \vec{p} + \alpha_p (\vec{q} - \vec{p})$$

$$\vec{q}' = \vec{q} - \alpha_q (\vec{q} - \vec{p})$$

$$\alpha_p = \frac{m_q}{(m_p + m_q)} \delta$$

$$\alpha_q = \frac{m_p}{(m_p + m_q)} \delta$$

$$m_p = m_q ?? \quad \frac{\delta}{2}$$



Cloth animation in action!

<https://philipclaude.github.io/csci461f25/chapter12/>

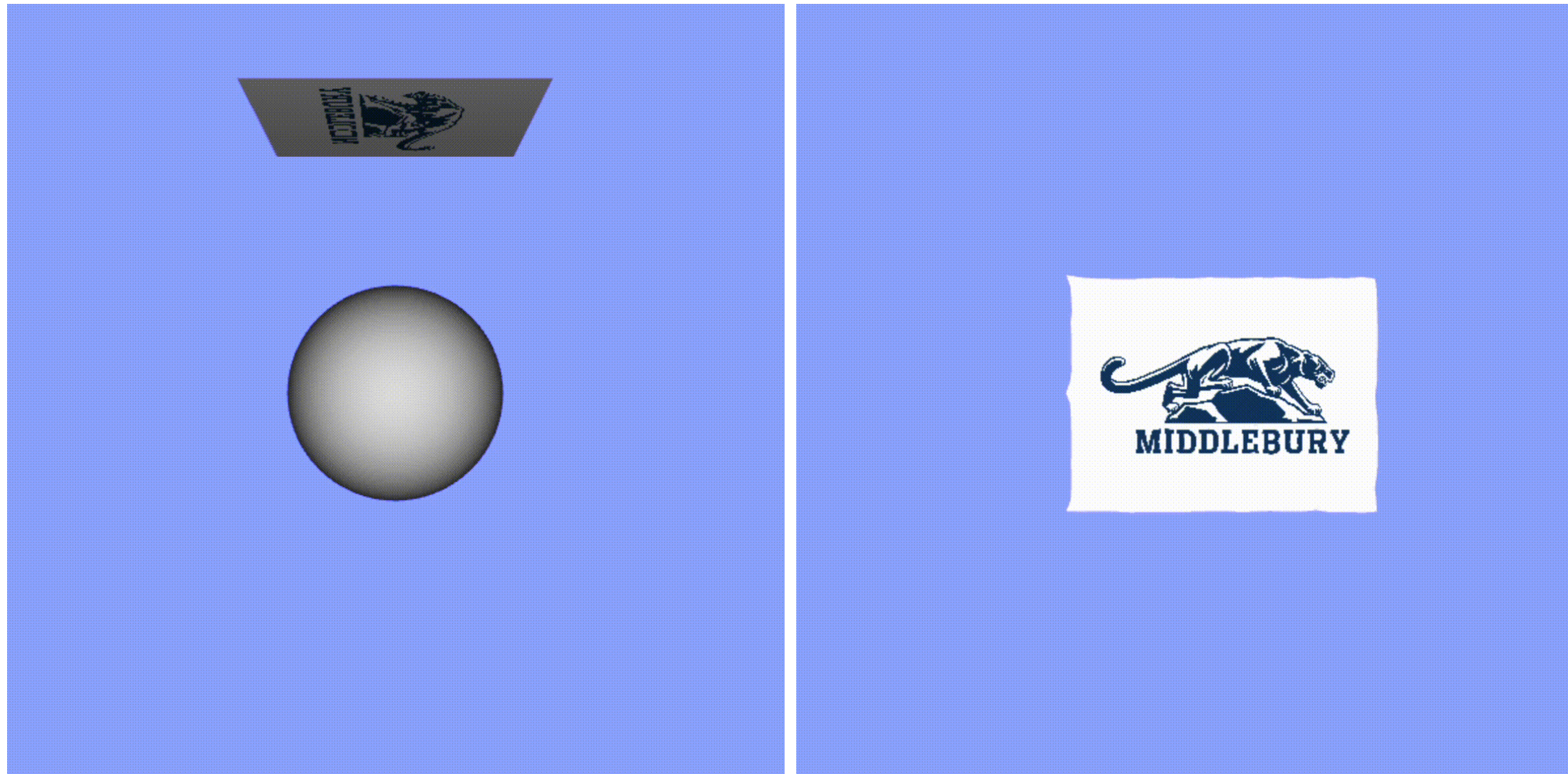
stage 1

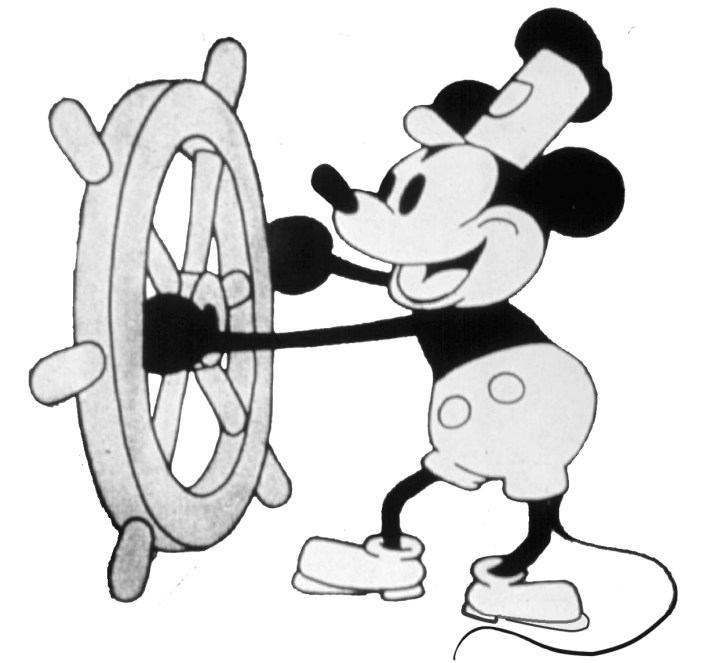
stage 2

```
1 // cloth update algorithm for one time step (frame)
2 for numIter iterations: // numIter is usually about 1 or 2
3     for each particle:
4         particle.forceUpdate();
5     for each constraint:
6         p = constraint.p; // first endpoint vertex (vec3)
7         q = constraint.q; // second endpoint vertex (vec3)
8         delta = (||q - p|| - L0) / ||q - p||; // vec3
9         dx = delta * (q - p); // vec3
10        mt = mp * mq / (mp + mq); // mp and mq are the masses of particles p and q
11        constraint.p += dx * mt / mp;
12        constraint.q -= dx * mt / mq;
```

Summary

- Cloth model involves **points** and **constraints**,
- Update points from **external forces** using **Verlet integration**,
- Update points from **constraint forces**,
- You'll implement these techniques in Thursday's lab.







CSCI 461: Computer Graphics



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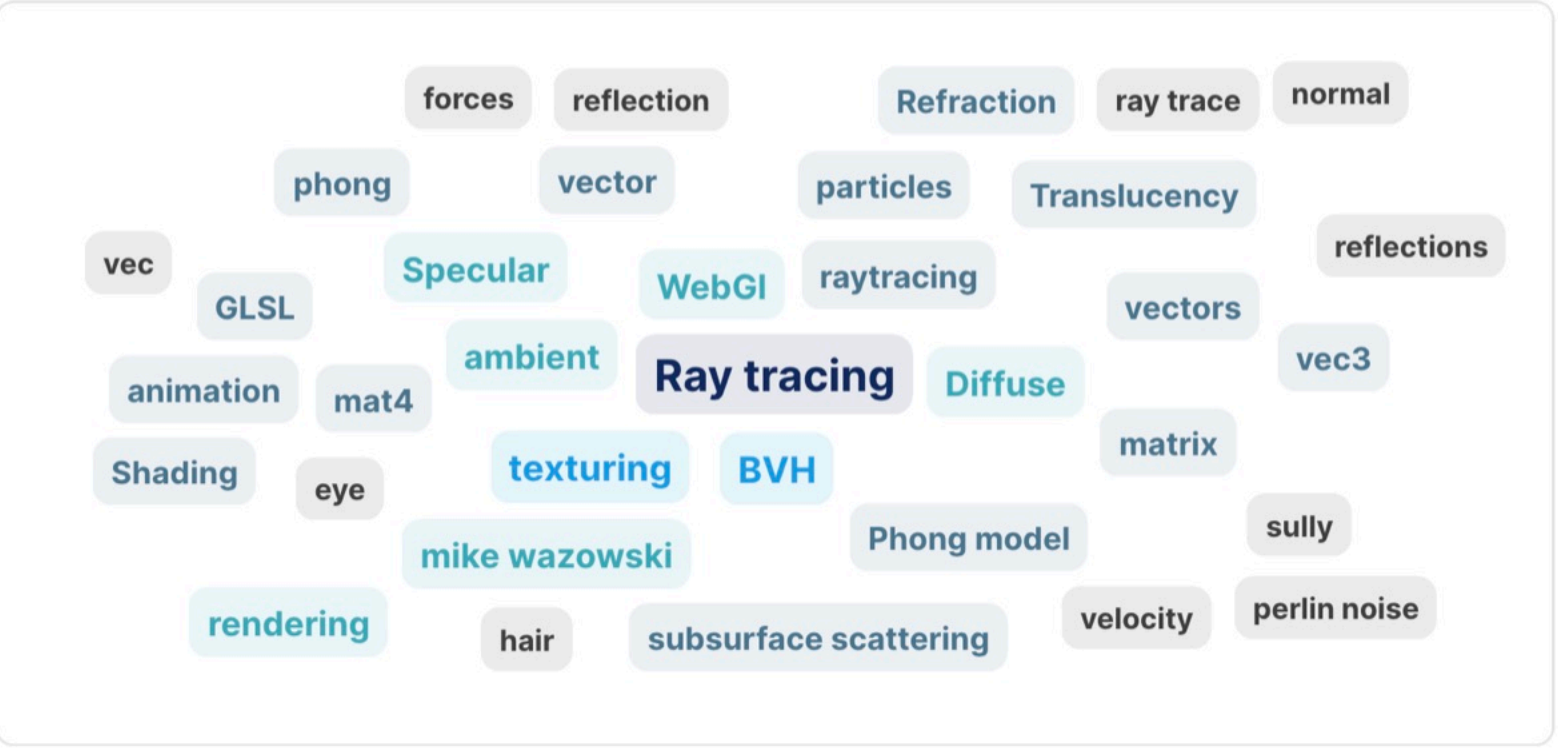
Course Recap



Exercise: list keywords from the course in groups of 2 (slido #3834500).

 cs461f25-lecture12 

 List some keywords from the semester! Include course topics or objects/scenes we rendered. 22 



A word cloud containing various computer graphics and course-related terms. The words are arranged in a roughly circular pattern. The most prominent words, shown in larger fonts, include 'Ray tracing', 'Diffuse', 'Specular', 'WebGL', 'texturing', 'BVH', 'rendering', 'ambient', 'GLSL', 'animation', 'Shading', 'eye', 'mat4', 'vec', 'phong', 'forces', 'reflection', 'Refractive', 'ray trace', 'normal', 'particles', 'Translucency', 'raytracing', 'vectors', 'reflections', 'vec3', 'matrix', 'sully', 'perlin noise', 'velocity', 'subsurface scattering', 'hair', 'mike wazowski', 'Phong model', and 'vectors'.

Add response

Where to go from here?



- Try other features from the labs,
- Check out other APIs and tools: [WebGPU](#), [OpenGL](#), [Shadertoy](#), [Three.js](#),
- Try implementing labs or Ray Tracing Book series in [C++](#).
- Become a [student volunteer](#) SIGGRAPH 2026! Applications due in February 2026.

Please complete the Course Response Form (this is the pre-lab this week).

[go/crf](#)

(keep screenshot of the confirmation page so you can submit the pre-lab)

Thank you for a fun semester!

Cloth setup for Lab 11.

