We know how to take pictures right now.

ray tracing or rasterization
(WebGL)

How should we create movies?

moving picture

today

after break

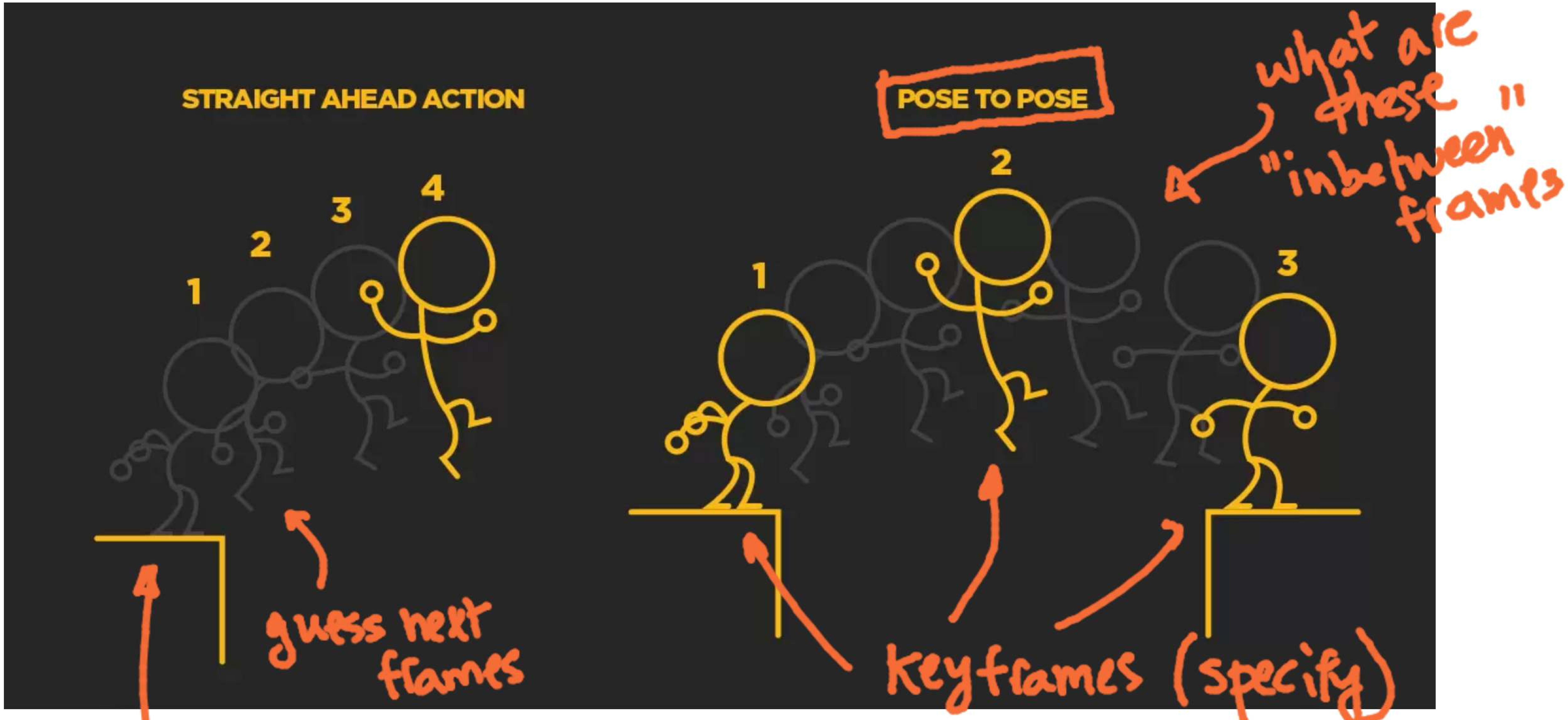
mesh

fluid

very hard to predict motion
use physics?

rigging
"skeleton" + "skin"

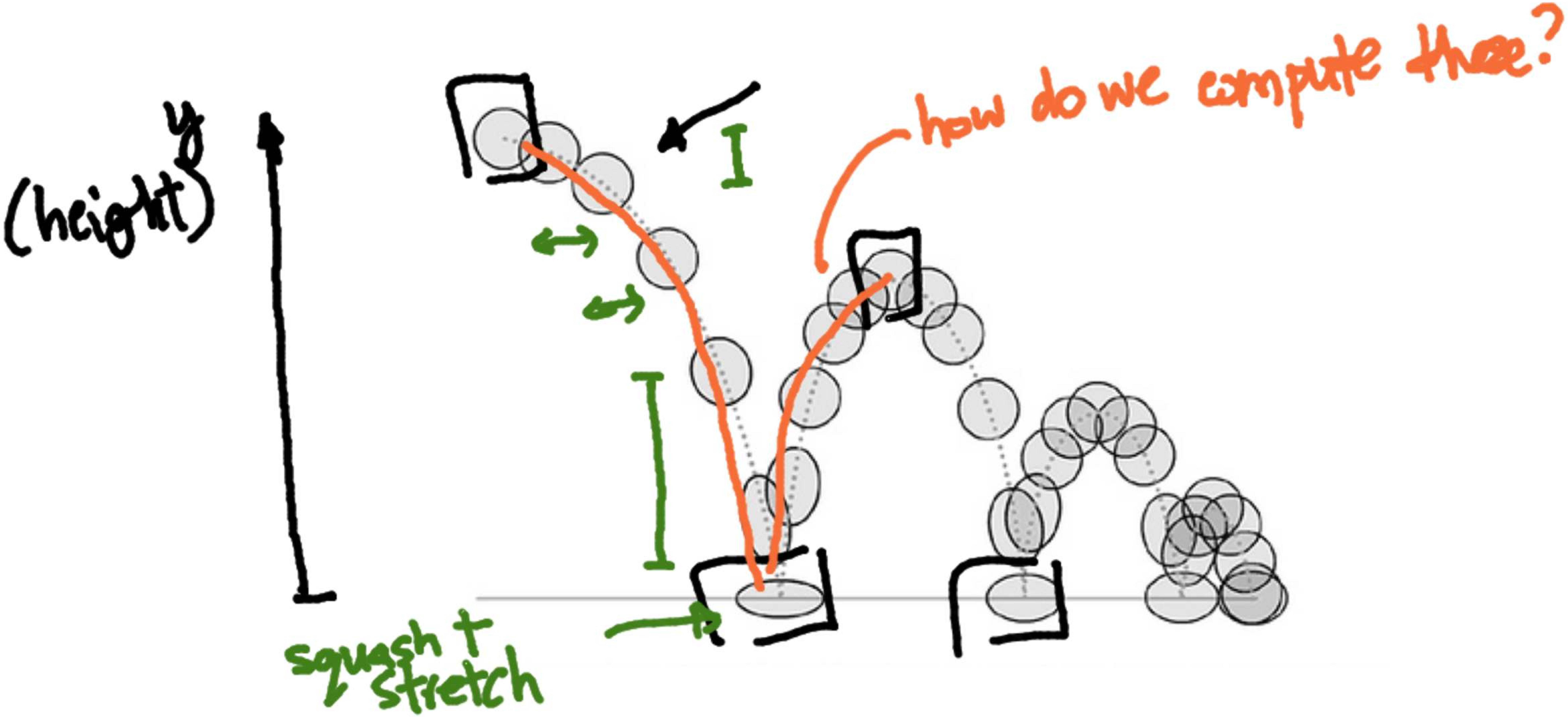
Another option? Let the artist decide.



specify initial frame

(source)

Main goal: compute "inbetween" frames by interpolating scene parameters at known frames.

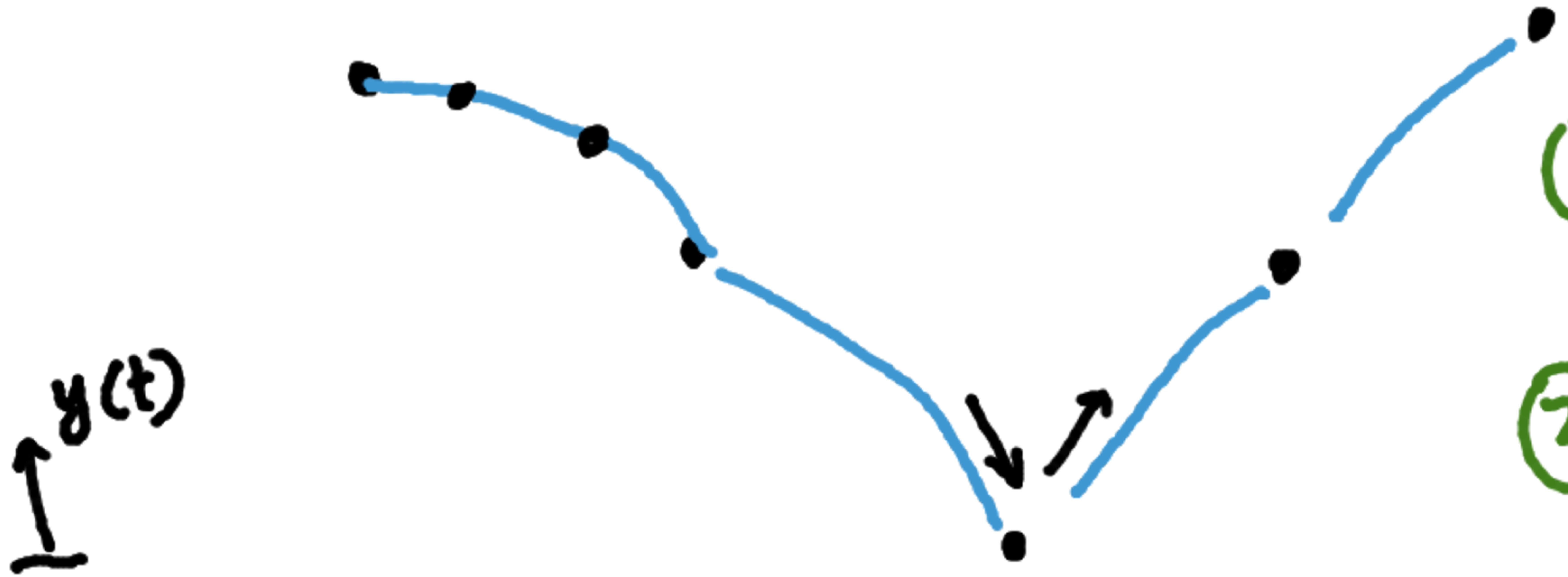


Other things to consider: squash & stretch, timing, slow in and out, arcs (Lasseter, 1987).

famous paper

Attempt #3: splines - glue together several curves.

cubic polynomials



1. specify additional points?
2. specify derivatives at end points
3. specify control points

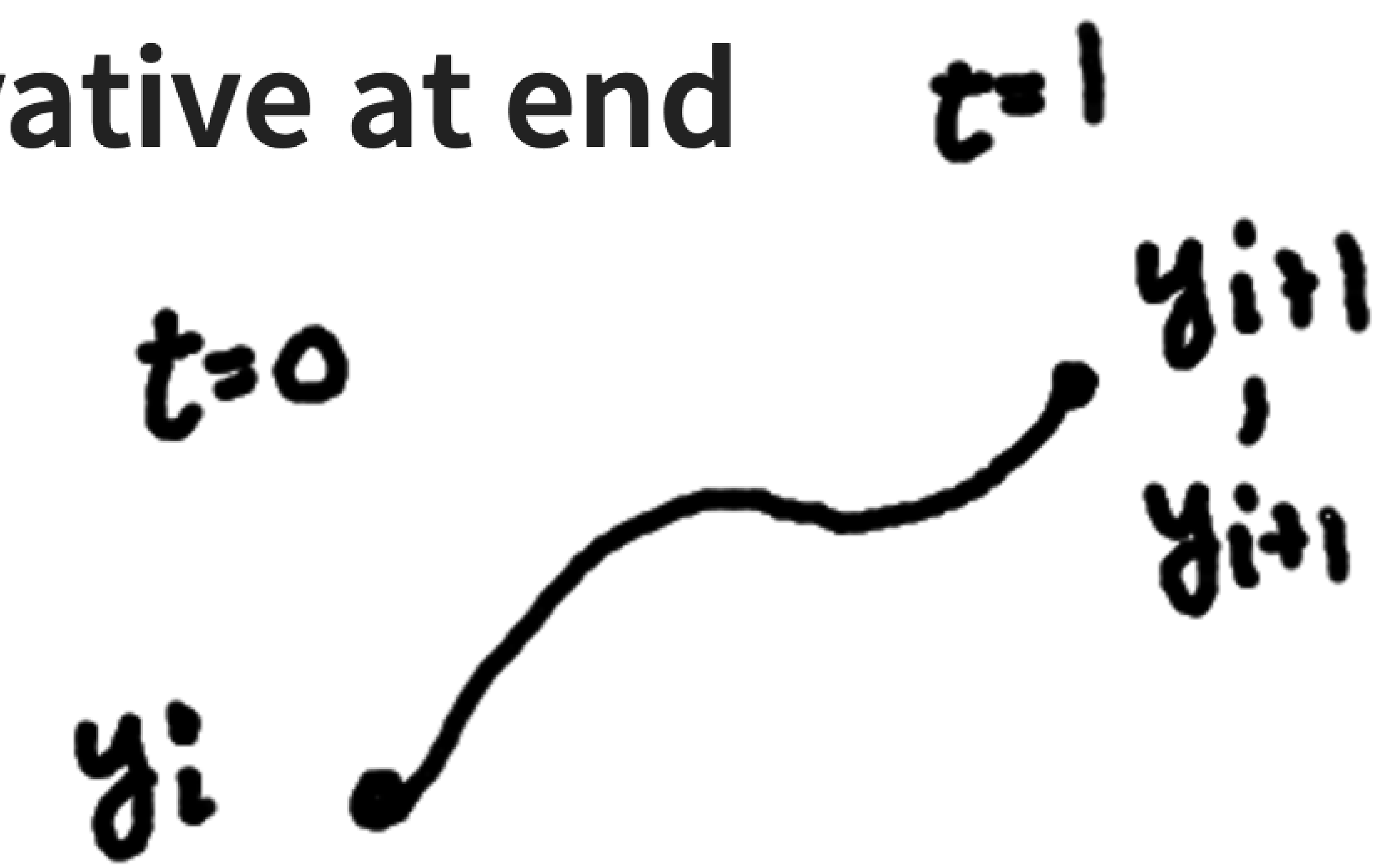
create a tool that's intuitive to use.

$$y(t) = c_0 + t c_1 + t^2 c_2 + t^3 c_3$$

$$y'(t) = \frac{dy}{dt} = c_1 + 2t c_2 + 3t^2 c_3$$

1. specify y at endpoints
2. specify y' at endpoints

Hermite basis: specify value and derivative at end points.



y_i' ← how quickly changes in time.

(a) $y(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$

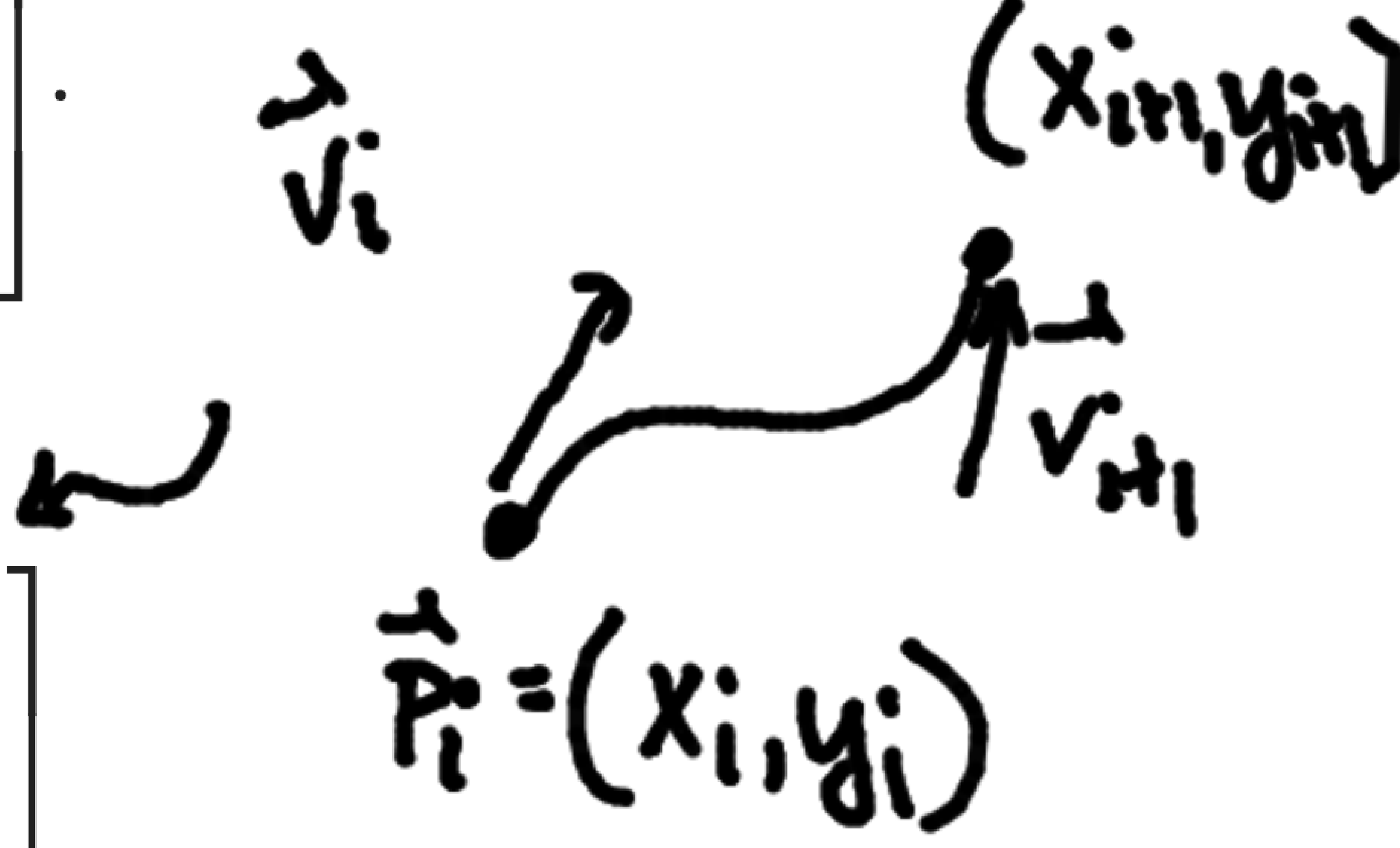
(b) $y'(t) = c_1 + 2c_2 t + 3c_3 t^2$

$$y_i = c_0$$
$$y_i' = c_1$$
$$y_{i+1} = c_0 + c_1 + c_2 + c_3$$
$$y_{i+1}' = c_1 + 2c_2 + 3c_3$$

$$\begin{bmatrix} y_i \\ y_i' \\ y_{i+1} \\ y_{i+1}' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

Solving for coefficients. (solve for c_0, c_1, c_2, c_3)

$$y(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_i \\ y_{i+1} \\ y'_i \\ y'_{i+1} \end{bmatrix}.$$



And in 2d:

$$\vec{p}(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \vec{p}_i \\ \vec{p}_{i+1} \\ \vec{v}_i \\ \vec{v}_{i+1} \end{bmatrix}$$

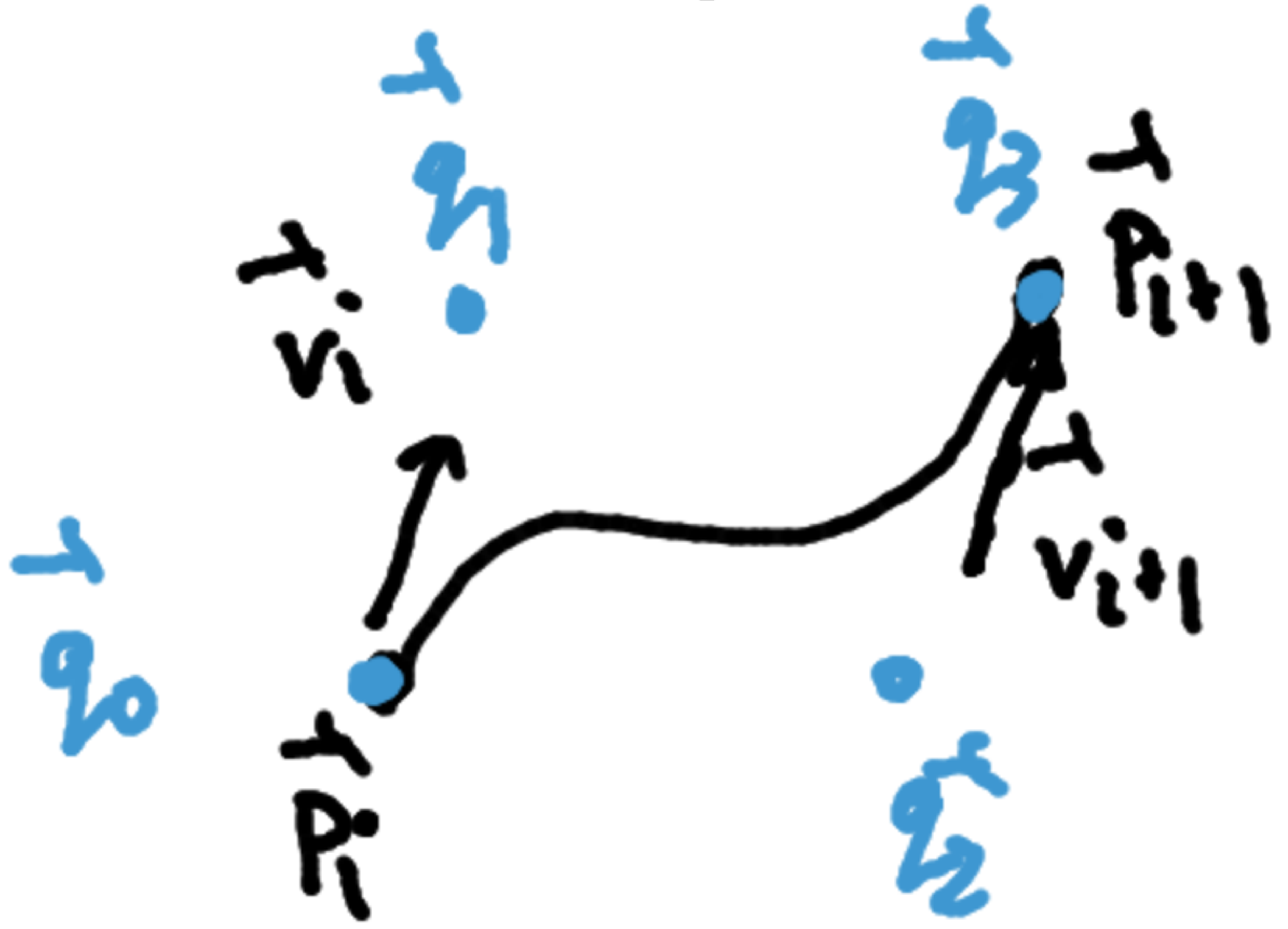
Hermite basis specify points + derivatives

not very intuitive to specify derivatives

Bézier basis: specify 4 controls points of each spline.

relate tangent vector $(\vec{v}_i, \vec{v}_{i+1})$

$$\boxed{\begin{array}{ll} \vec{q}_0 = \vec{p}_i & \vec{q}_3 = \vec{p}_{i+1} \\ \vec{v}_i = 3(\vec{q}_1 - \vec{q}_0) & \vec{v}_{i+1} = 3(\vec{q}_3 - \vec{q}_2) \end{array}}$$



$$\vec{p}(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \vec{p}_i \\ \vec{p}_{i+1} \\ \vec{v}_i \\ \vec{v}_{i+1} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} \vec{q}_0 \\ \vec{q}_1 \\ \vec{q}_2 \\ \vec{q}_3 \end{bmatrix}$$

Combining terms to isolate Bézier basis functions.

$$p(t) = \underline{\begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix}} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \vec{q}_0 \\ \vec{q}_1 \\ \vec{q}_2 \\ \vec{q}_3 \end{bmatrix}$$

$$p(t) = (-t^3 + 3t^2 - 3t + 1)\vec{q}_0 + (3t^3 - 6t^2 + 3t)\vec{q}_1 + (-3t^3 + 3t^2)\vec{q}_2 + (t^3)\vec{q}_3$$

$$= \underbrace{(1-t)^3}_{b_0(t)}\vec{q}_0 + \underbrace{3t(1-t)^2}_{b_1(t)}\vec{q}_1 + \underbrace{3t^2(1-t)}_{b_2(t)}\vec{q}_2 + \underbrace{t^3}_{b_3(t)}\vec{q}_3$$

Bézier basis functions

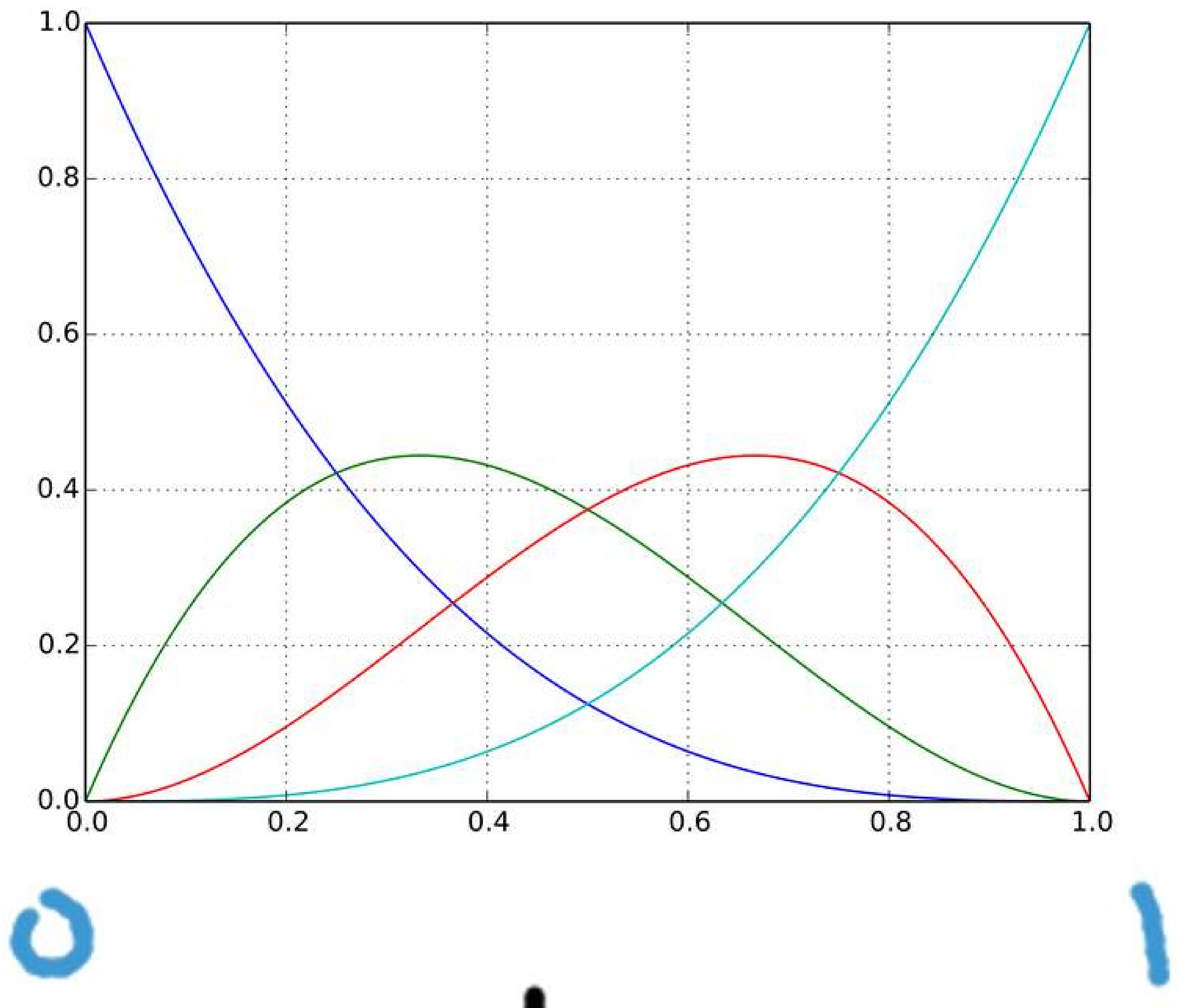
Properties of Bézier basis functions.

$$b_0(t) = (1-t)^3$$

$$b_1(t) = 3t(1-t)^2$$

$$b_2(t) = 3t^2(1-t)$$

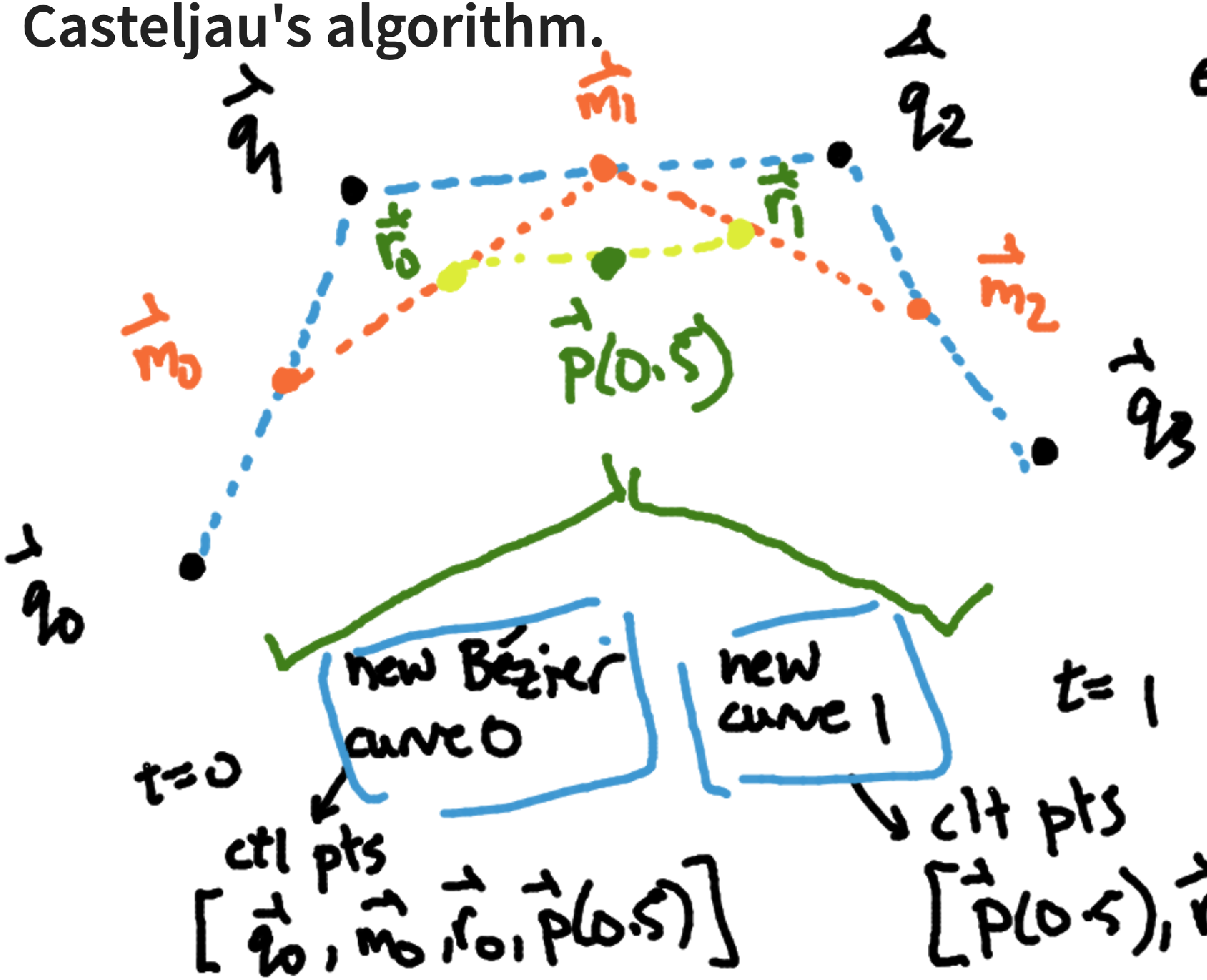
$$b_3(t) = t^3$$



$$b_0(t) + b_1(t) + b_2(t) + b_3(t) = 1$$

$$b_i(t) \geq 0 \quad 0 \leq i \leq 3$$

Evaluating a Bézier spline: use basis functions or de Casteljau's algorithm.



evaluate at $t=0.5$

not $t=0.5$?

$$\vec{m}_0 = \vec{q}_0 + t(\vec{q}_1 - \vec{q}_0)$$

....

$$\vec{r}_0 = \vec{m}_0 + t(\vec{m}_1 - \vec{m}_0)$$