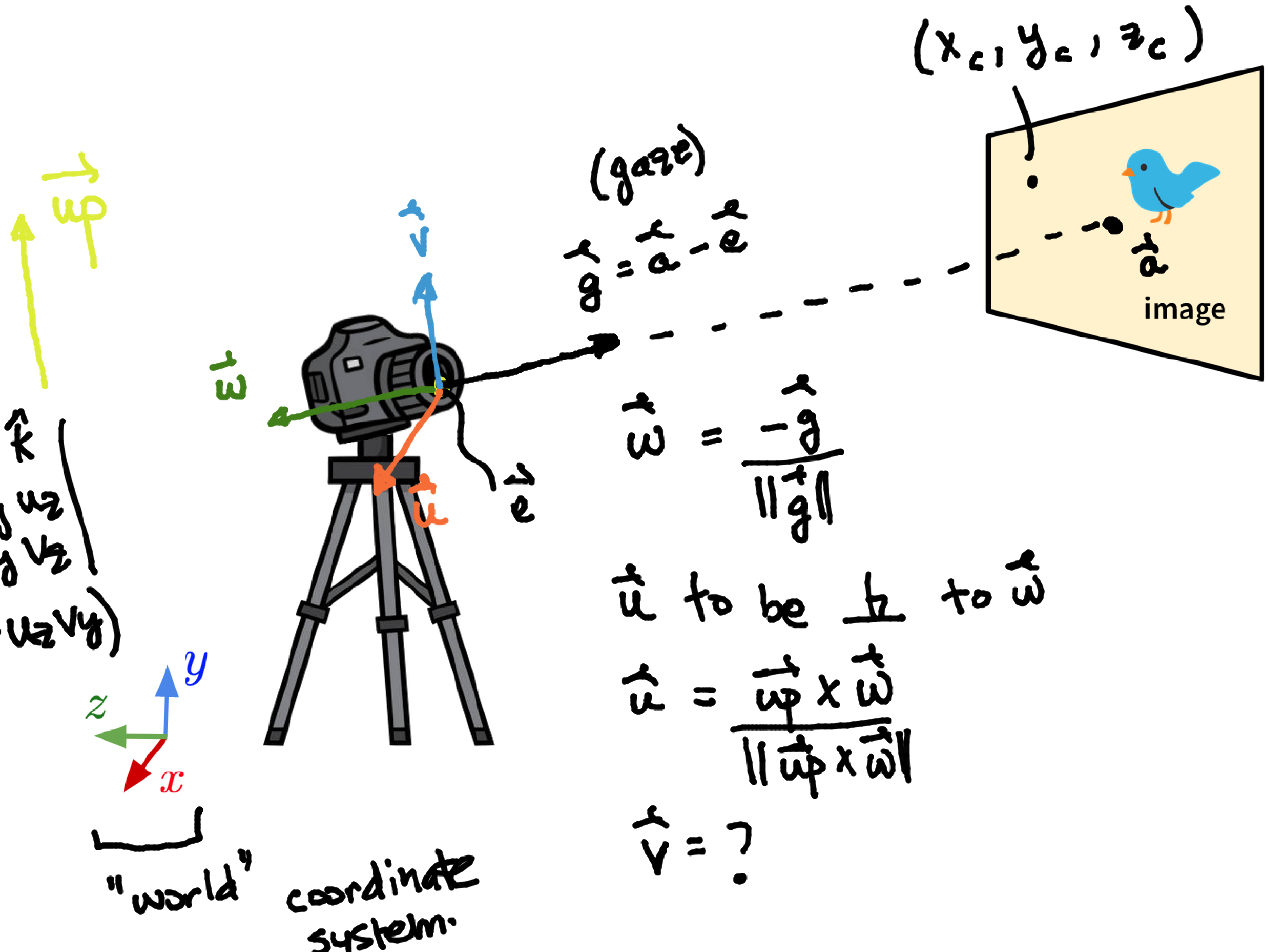
Pointing the camera at some point to "look at."



$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$$

$\hat{i}(u_y v_z - u_z v_y)$
$+\hat{j}\ldots$
$+\hat{k}\ldots$

$$\hat{w} = \frac{-\hat{g}}{\|\vec{g}\|}$$

$\hat{u}$ to be $\perp$ to $\hat{w}$

$$\hat{u} = \frac{\vec{up} \times \vec{w}}{\|\vec{up} \times \vec{w}\|}$$

$\hat{v} = ?$

What is the ray direction then?

$$\vec{x}(t) = \vec{e} + \underbrace{\vec{r}}_{\text{what is this?}} \cdot t$$

knowing change-of-basis from camera to world with $\vec{u}, \vec{v}, \vec{w}$.

$$\vec{q} = \vec{e} + x_c \vec{u} + y_c \vec{v} + z_c \vec{w}$$

3d pixel coords

$\frac{(i+0.5)}{n_x} \ldots$ $\quad (j+0.5) \ldots$ $\quad -d$

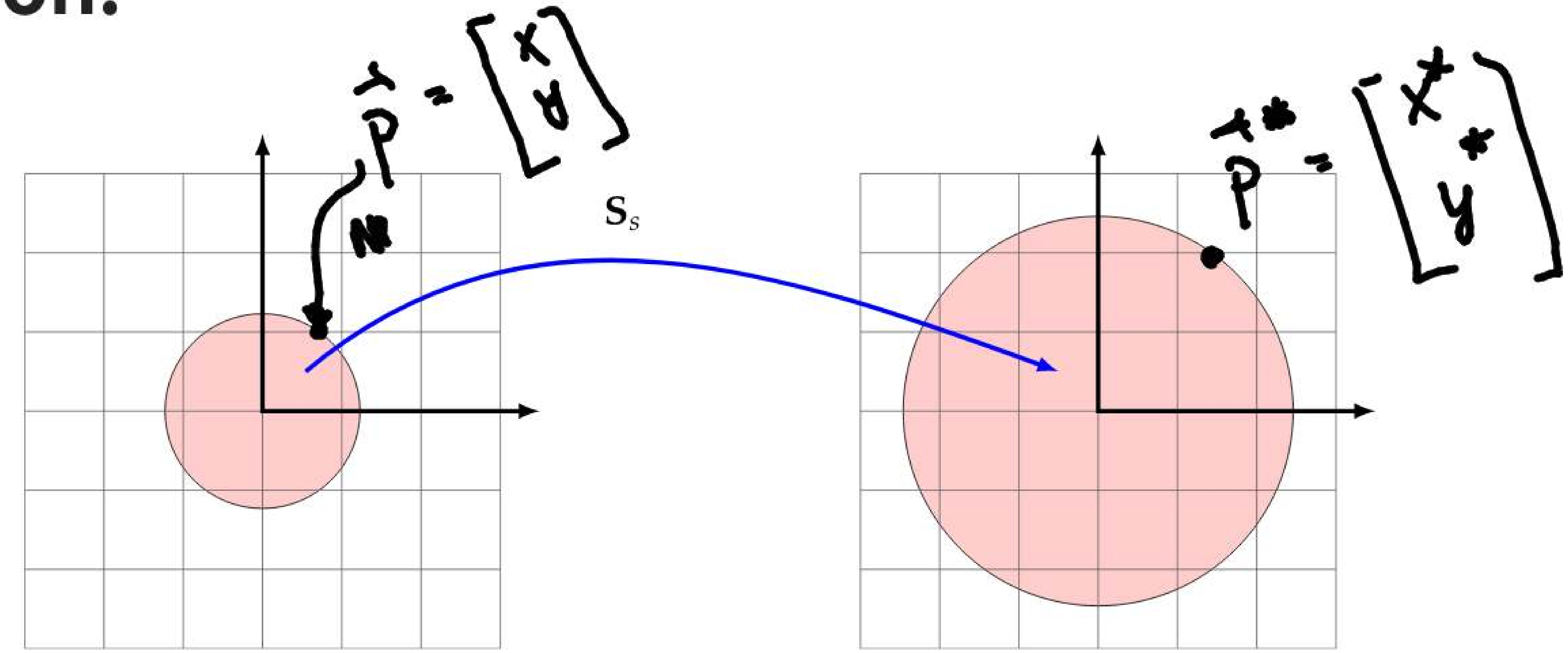
gl-matrix

$\boxed{\text{targetTo}}$

$$\vec{r} = \vec{q} - \vec{e} = x_c \vec{u} + y_c \vec{v} + z_c \vec{w}$$

$$= \underbrace{\begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix}}_{B} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

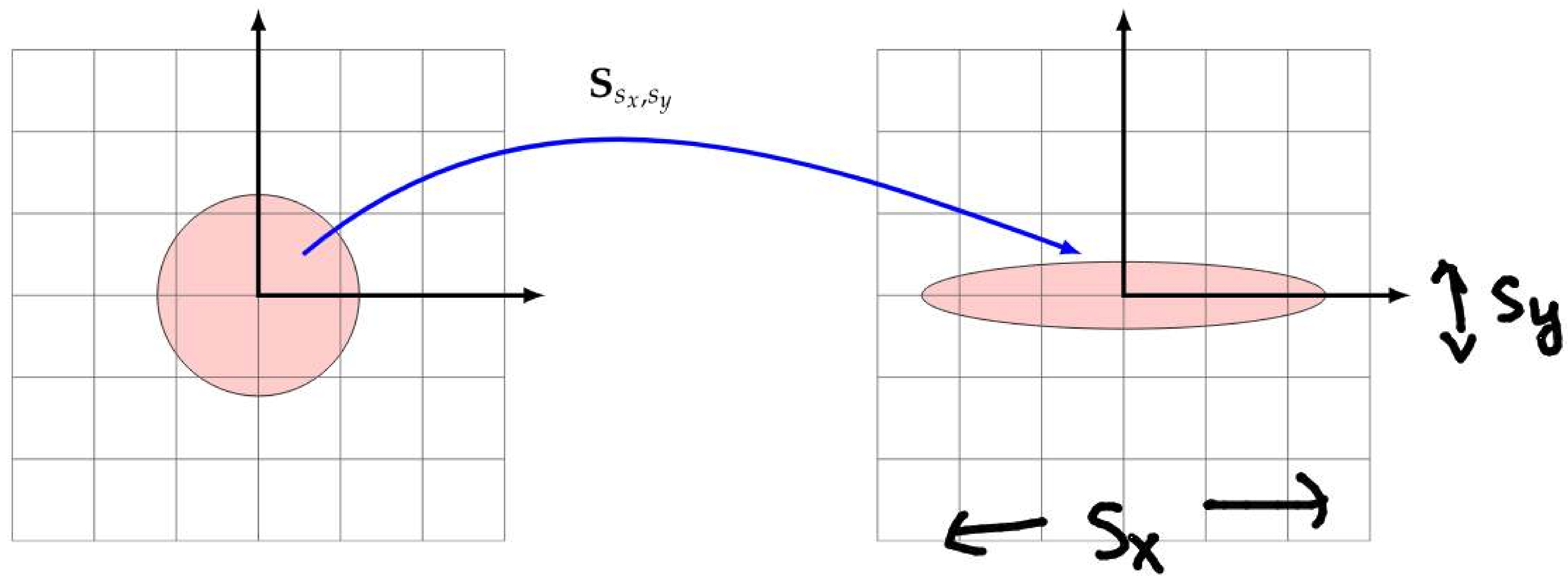
Scaling transformation stretches points in each dimension.



$$x^* = s \cdot x \qquad = s \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$
$$y^* = s \cdot y$$

$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling transformation stretches points in each dimension (can be non-uniform).



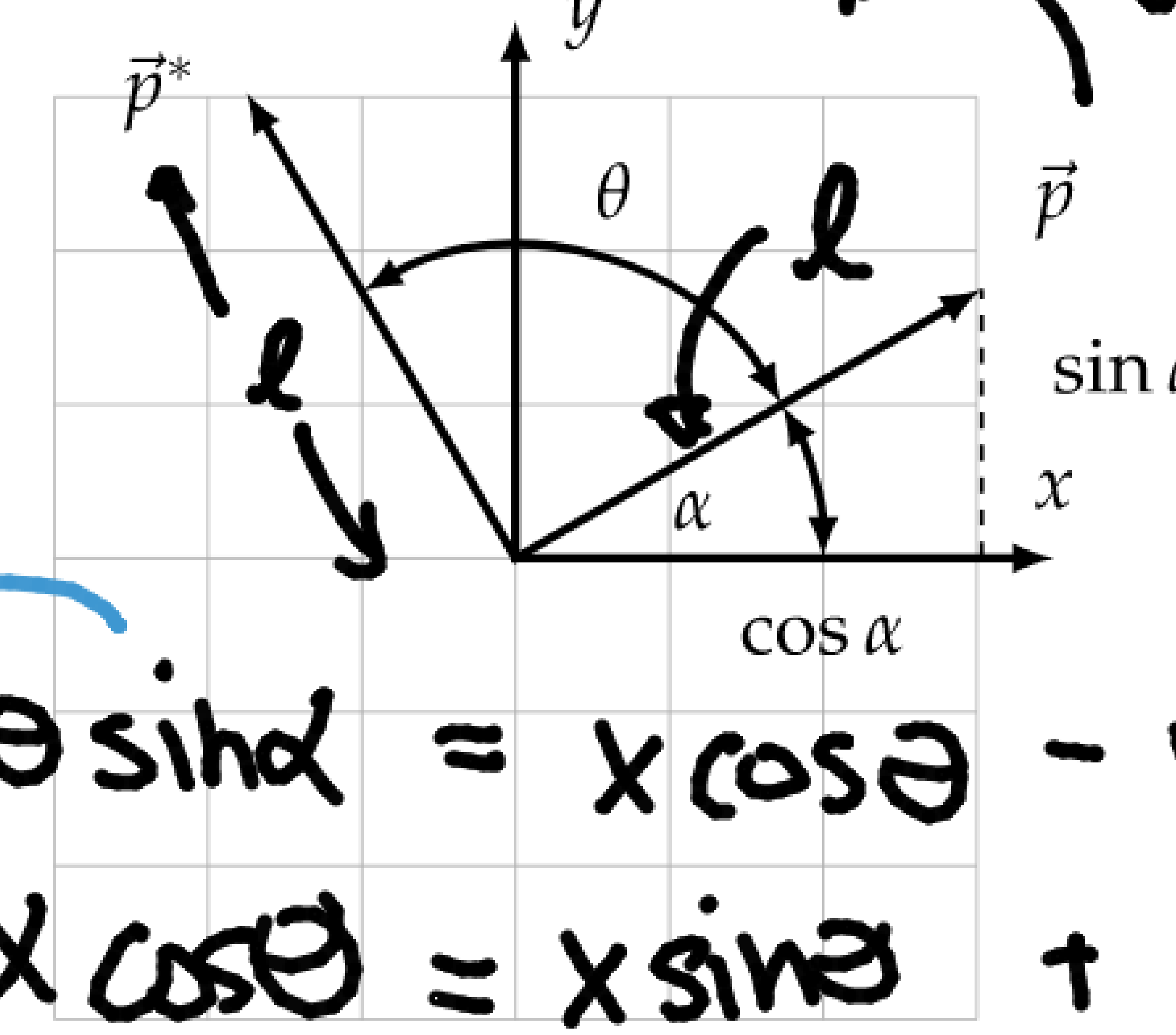
$$x^* = s_x \cdot x$$
$$y^* = s_y \cdot y$$

$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \underbrace{\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}}_{\text{Scaling matrix}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotation matrix rotates vectors by an angle about some axis.

$$\vec{p} = (x, y) \qquad \boxed{\begin{aligned} x &= \ell\cos\alpha \\ y &= \ell\sin\alpha \end{aligned}}$$

rotate by angle θ



calculate $\vec{p}^* = (x^*, y^*)$ after rotation

$$x^* = \ell\cos(\theta + \alpha) = \ell\cos\theta\cos\alpha - \ell\sin\theta\sin\alpha = x\cos\theta - y\sin\theta$$

$$y^* = \ell\sin(\theta + \alpha) = \ell\sin\theta\cos\alpha + \ell\sin\alpha\cos\theta = x\sin\theta + y\cos\theta$$

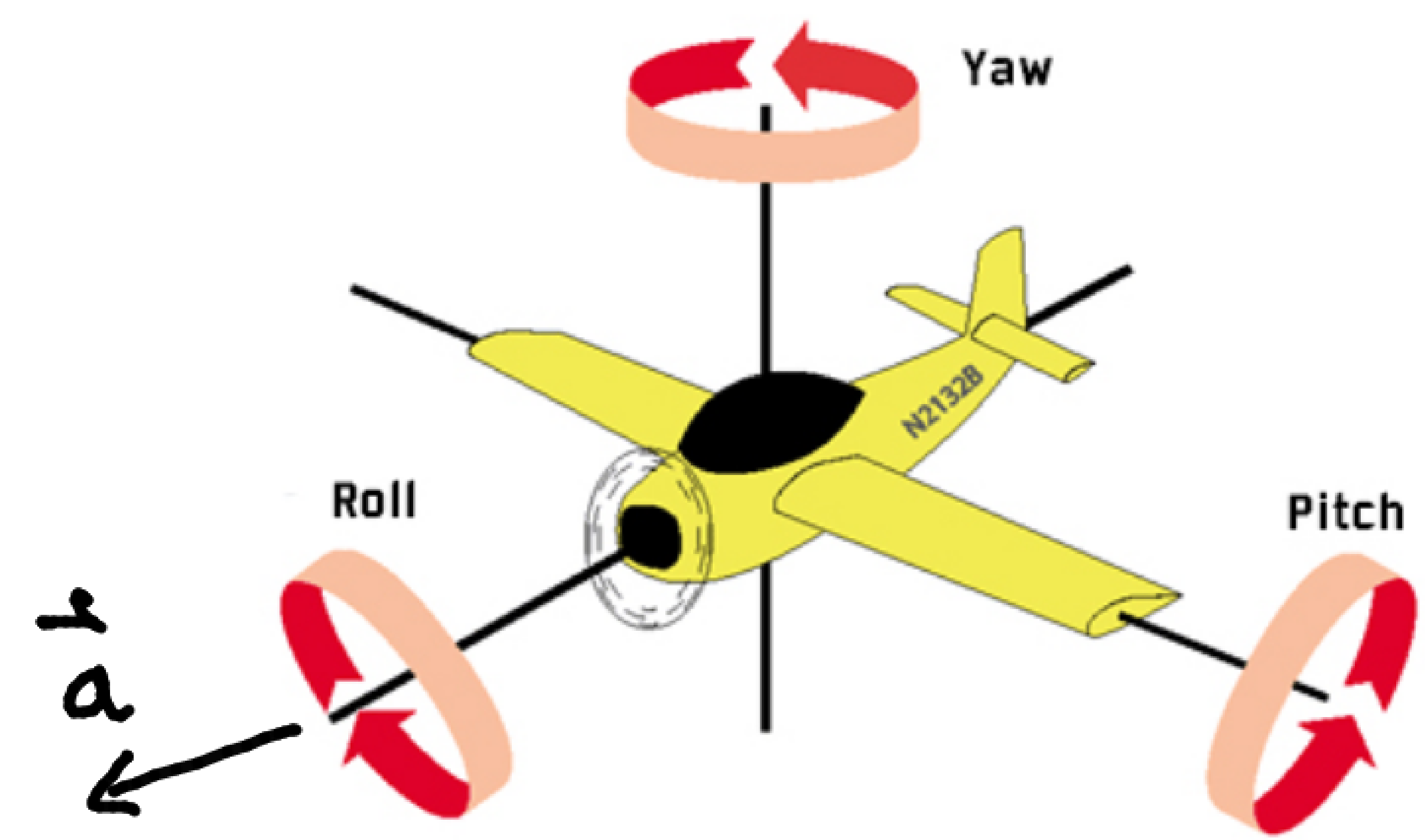
$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotations about the x, y and z-axes.

$$\mathbf{R}_{\theta,x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_{\theta,y} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_{\theta,z} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$R_z^{2d} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$-\theta$

$$R^{-1} = \frac{1}{\underbrace{\cos^2\theta + \sin^2\theta}_{1}} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

When you substitute -theta into R (pick either Rx, Ry, or Rz), what do you obtain? 17

(a) transpose(R)

(b) inverse(R)

(c) Both (a) and (b)

Voting as Anonymous

Send

$$R^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

How to represent a translation (shift) with a matrix?

$$\vec{p}^{\,*} = \vec{p} + \underbrace{\vec{t}}_{\text{shift}} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$$

$\begin{bmatrix} x \\ y \end{bmatrix}$

$$\cancel{\vec{p}^{\,*} = \underbrace{\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix}}$$

we can't!

what about this?

$$\vec{p}^{\,*} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

→ represent points using homogeneous coords!

Homogeneous coordinates is a trick to combine everything into a single matrix.

Main idea: introduce new coordinate equal to 1 for points.

$\vec{p}^{\,h} \to$ homogeneous

$$\vec{p}^{\,h} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

4d vectors

$\vec{p}, \vec{q} \in \mathbb{R}^3$

what is $\hat{v} = \hat{q} - \hat{p}$ using homogeneous coordinates?

$$\hat{v} = \begin{bmatrix} q_x - p_x \\ q_y - p_y \\ q_z - p_z \\ 1 - 1 \end{bmatrix}$$

0

Combining transformations: read from right-to-left!

$$\vec{p} \rightarrow \vec{p}^{*} = \underbrace{M_1}_{1^{st}\ \text{transformation}} \vec{p}$$

$$\vec{p}^{**} = M_2 \underbrace{\left(M_1 \vec{p}\right)}_{\vec{p}^{*}}$$

$$\cdots = M_n \cdots \left(M_3 \underbrace{\left(M_2 \vec{p}^{*}\right)}_{\vec{p}^{**}}\right) \quad (M_1 \vec{p}) \quad \underbrace{\left(M_n \cdots M_3\, M_2\, M_1\right)}_{\text{compound transformation } M} \vec{p}$$

Transform normals using the inverse-transpose of your transformation.

$M^T N = I \rightarrow \boxed{N = (M^T)^{-1}}$

before

$\vec{n} \cdot \vec{t} = 0$

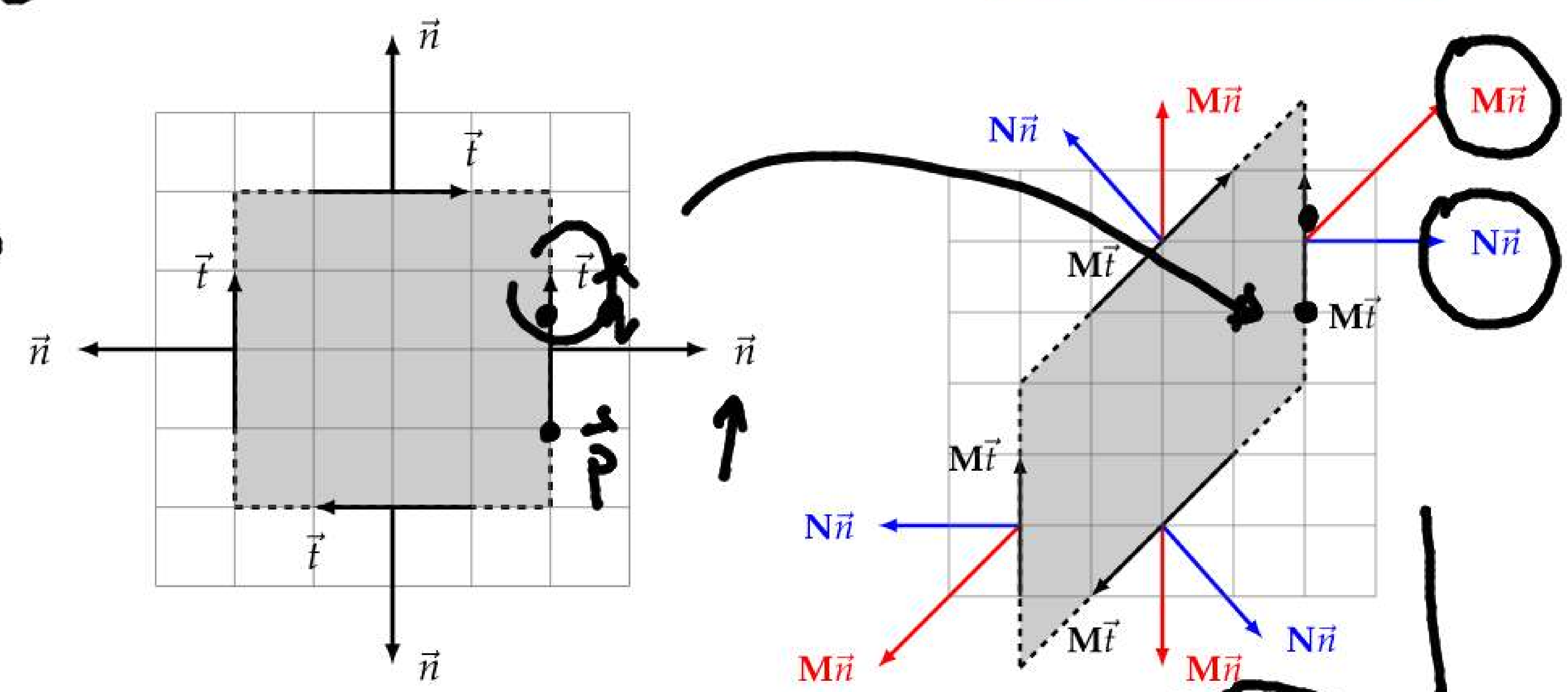
$\hookrightarrow \hat{n}^T \hat{t} = 0$

after: $\vec{t}^*, \vec{n}^*$

$\vec{t}^* = M\vec{t}$

$\vec{n}^* = N\hat{n}$

↑ what is this?



transformation M

(1) $\vec{t}$ also transformed by M.

(2) normals are ⊥ to surface before & after

$\vec{t}^{*T}\vec{n} = (M\vec{t})^T(N\vec{n}) = 0$

$= \boxed{\vec{t}^T M^T N \hat{n} = 0}$

↑ identity