Middlebury

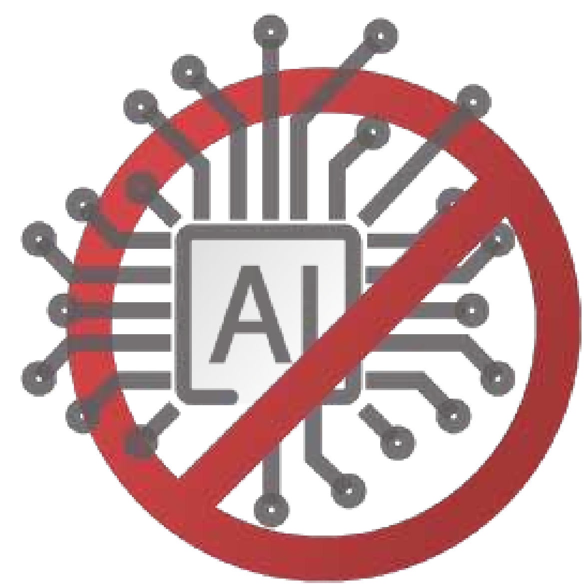
CSCI 201: Data Structures

Fall 2024

Lecture 11R: Graph Searching

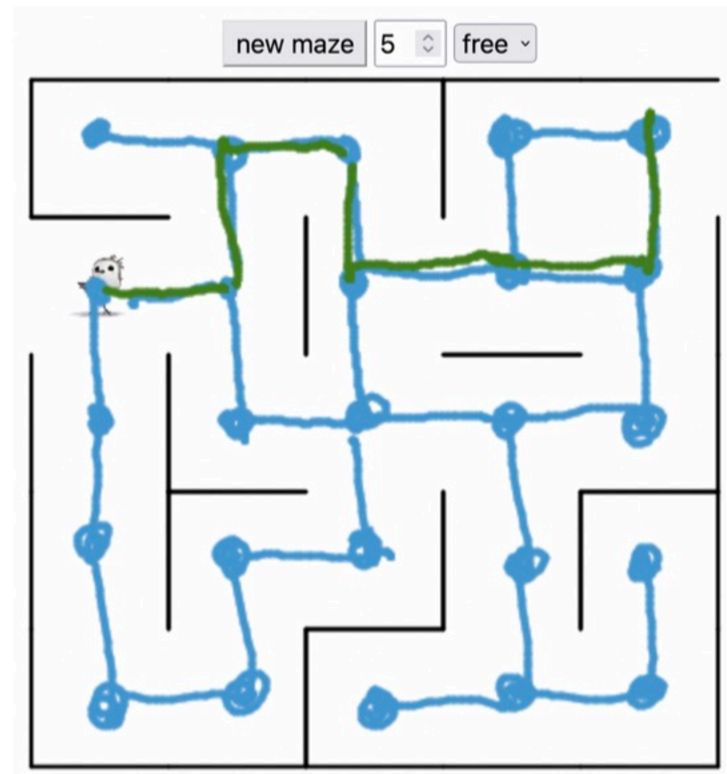
Final Reminder: You are NOT allowed to use generative AI to generate code for you in ANY way.

- The use of generative AI appears to be increasing (through Homework 7 and Homework 8).



- Homeworks 9 & 10, and Final Exam suspected of generative AI use will be assigned a grade of **ZERO** (with an option to meet with me to discuss whether I made a mistake in suspecting AI use).
- If you already used generative AI for Homework 9, redo the assignment from scratch.

Before we jump into our goals for today:

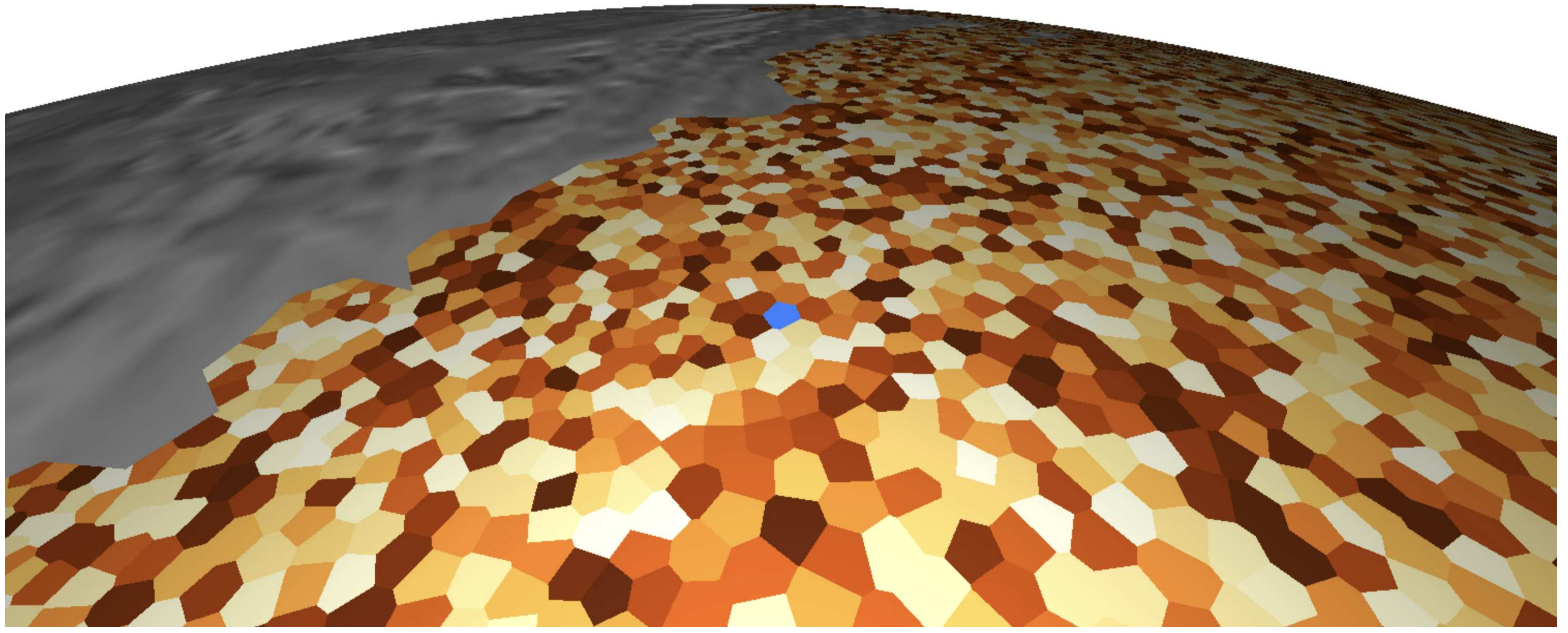


click on "game" in the row for today's class at go/cs201

What is the relationship to graphs? What are the nodes? What are the edges?

Goals for today:

- Visit nodes in a graph using Depth-First Search (DFS).
- Visit nodes in a graph using Breadth-First Search (BFS).
- Implement DFS and BFS.



Very common topics in tech interviews.

Let's revisit our getEdges method from last class.

```java
public class Graph<Node> {

  Set<Edge> getEdges() {
    Set<Edge> edges = new HashSet<>();

    // loop through all nodes in the graph
    for (Node u : adj.keySet()) {

      // loop through all nodes adjacent to node u
      for (Node v : adj.get(u)) {

        // don't double-count this edge
        Edge edge = new Edge(u, v);
        if (!edges.contains(edge)) {
          edges.add(edge);
        }
      }
    }
    return edges;
  }
}
```

O(1)

eval. hash function + potentially handle collisions

```java
public class Graph<Node> {

  List<Edge> getEdges() {
    List<Edge> edges = new ArrayList<>();

    // loop through all nodes in the graph
    for (Node u : adj.keySet()) {

      // loop through all nodes adjacent to node u
      for (Node v : adj.get(u)) {

        // don't double-count this edge
        Edge edge = new Edge(u, v);
        if (!edges.contains(edge)) {
          edges.add(edge);
        }
      }
    }
    return edges;
  }
}
```

O(n)

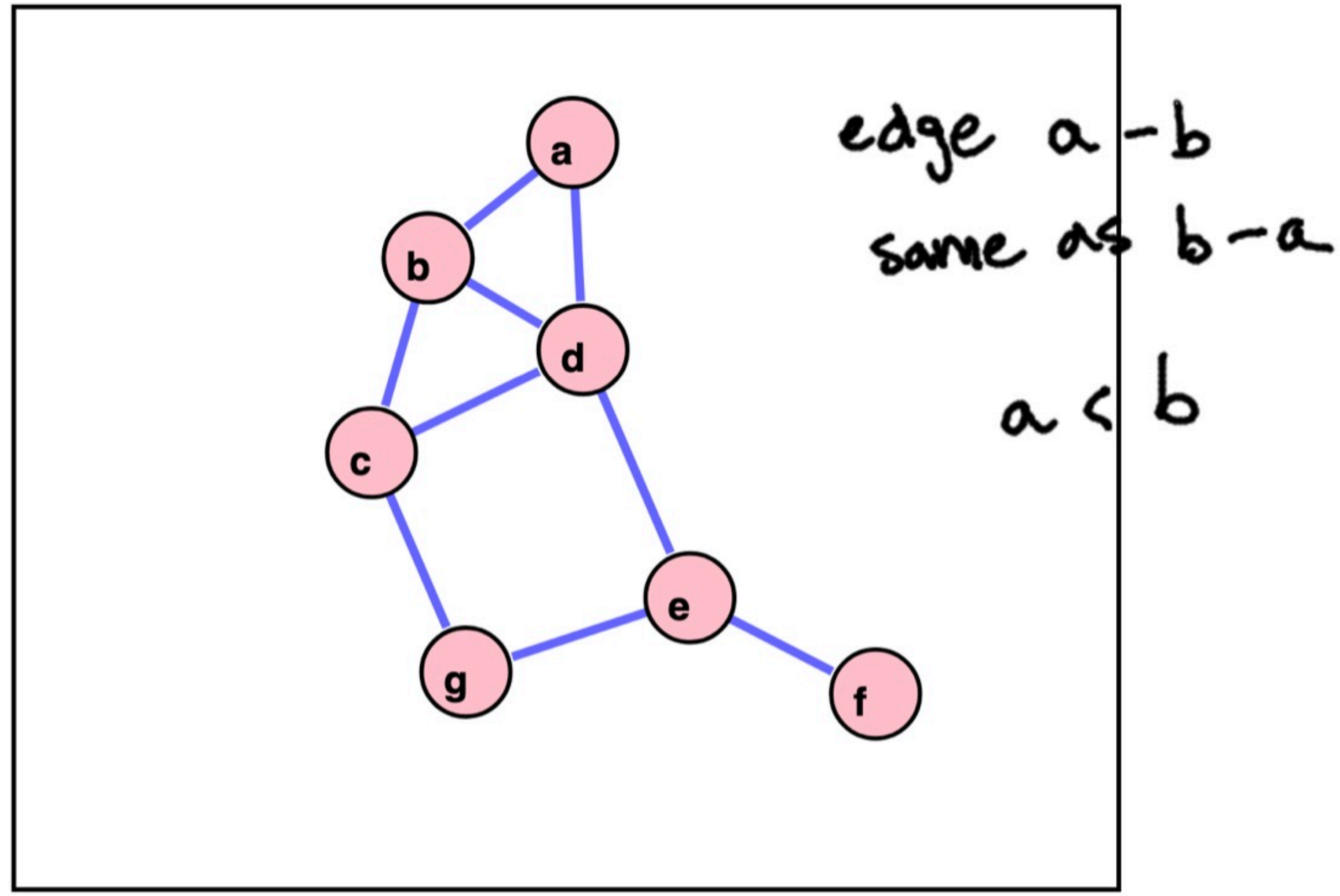
```java
// means we needed
class Edge {

  public int hashCode() {
    // edges (u, v) and (v, u)
    // should have the same hash table index
    ...
  }

  public boolean equals(Object otherObj) {
    // edges (u, v) and (v, u)
    // should be considered equal
    ...
  }
}
```

```java
// then we just need
class Edge {

  public boolean equals(Object otherObj) {
    // edges (u, v) and (v, u)
    // should be considered equal
    ...
  }
}
```

Another option if Node is Comparable.

```java
public class Graph<Node extends Comparable<Node>> {

  List<Edge> getEdges() {
    List<Edge> edges = new ArrayList<>();

    // loop through all nodes in the graph
    for (Node u : adj.keySet()) {

      // loop through all nodes adjacent to node u
      for (Node v : adj.get(u)) {

        // since adj(u) stores v
        // and adj(v) stores u
        if (u.compareTo(v) < 0) {
          edges.add(new Edge(u, v));
        }
      }
    }
    return edges;
  }
}
```

```java
// then we just need
class Edge {
  public Node u;
  public Node v;

  public Edge(Node u, Node v) {
    this.u = u;
    this.v = v;
  }
}
```



Consider three variants for storing adjacent nodes.

What is the complexity of checking if a node u is adjacent to v?

```java
public class Graph<Node> {

  HashMap<Node, TreeSet<Node>> adj;

  void addEdge(Node a, Node b) {
    if (!adj.containsKey(a)) {
      adj.put(a, new TreeSet<>());
    }
    if (!adj.containsKey(b)) {
      adj.put(b, new TreeSet<>());
    }
    adj.get(a).add(b);
    adj.get(b).add(a);
  }

  boolean areAdjacent(Node a, Node b) {
    return adj.get(a).contains(b);
  }
}
```

TreeSet (BST)

advantage: predictable order of adjacent nodes.

disadvantage: O(log n) → for contains

↑ but not really so bad

n ↑ # adjacent nodes

```java
public class Graph<Node> {

  HashMap<Node, HashSet<Node>> adj;

  void addEdge(Node a, Node b) {
    if (!adj.containsKey(a)) {
      adj.put(a, new HashSet<>());
    }
    if (!adj.containsKey(b)) {
      adj.put(b, new HashSet<>());
    }
    adj.get(a).add(b);
    adj.get(b).add(a);
  }

  boolean areAdjacent(Node a, Node b) {
    return adj.get(a).contains(b);
  }
}
```

Hashset (hash table)

advantage: O(1) for contains

disadvantage: not predictable.

```java
public class Graph<Node> {

  HashMap<Node, ArrayList<Node>> adj;

  void addEdge(Node a, Node b) {
    if (!adj.containsKey(a)) {
      adj.put(a, new ArrayList<>());
    }
    if (!adj.containsKey(b)) {
      adj.put(b, new ArrayList<>());
    }
    adj.get(a).add(b);
    adj.get(b).add(a);
  }

  boolean areAdjacent(Node a, Node b) {
    return adj.get(a).contains(b);
  }
}
```

ArrayList

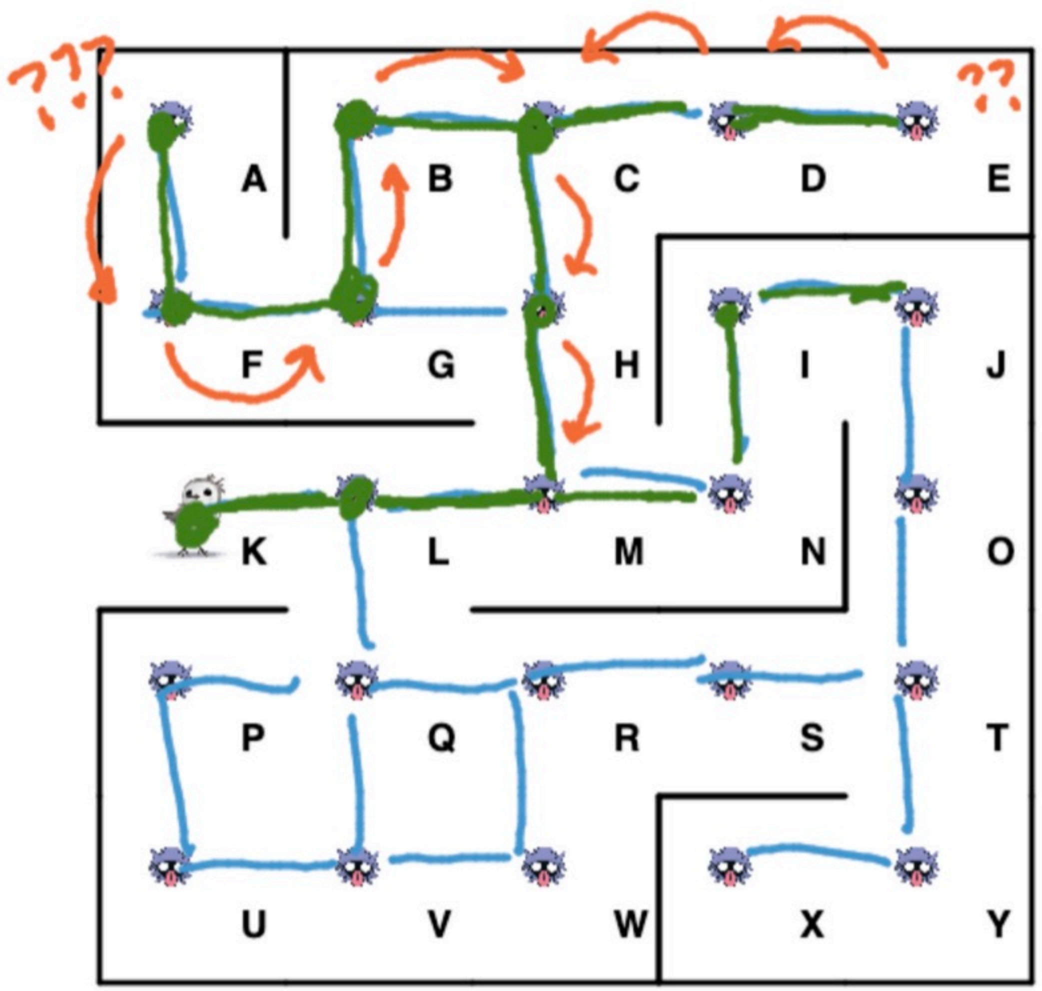
advantage: predictable order but depends on order of addEdge

disadvantage: O(n) for contains

Depth-First Search ("backtracking").

assume adjacent nodes visited in alphabetical order

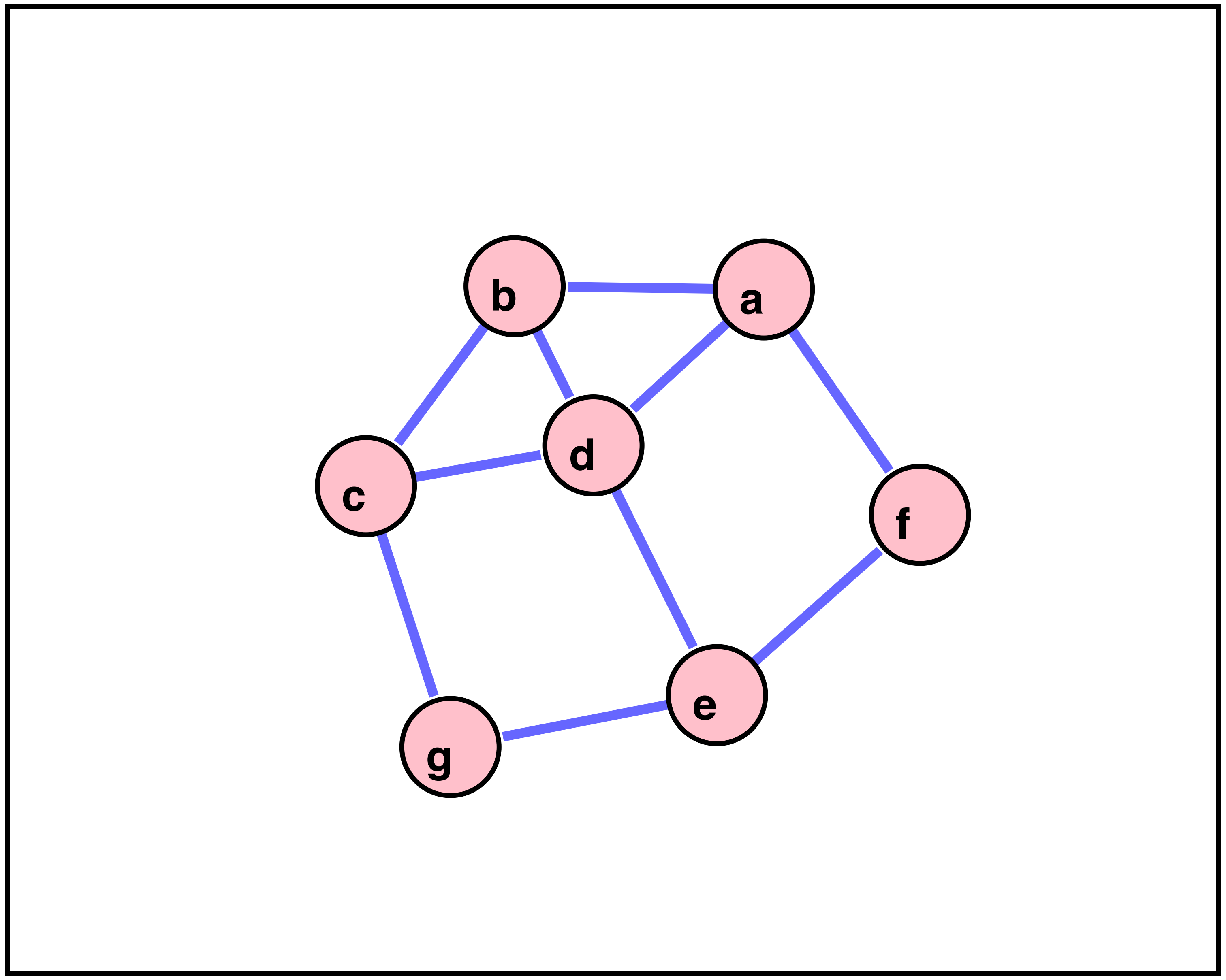
- **Main idea:** Keep traversing edges until you "hit a wall," then go back to parent.
- Don't step into nodes we already visited.
- Resulting set of edges forms a **tree: connected** and **acyclic**



K - L - M - H - C - B - G - F - A - D - E - N - I - J ...

Exercise: visit all nodes using DFS, starting at **b**. List the order in which nodes are traversed.

Assume we are using `TreeSet<Node>` to store adjacent nodes.

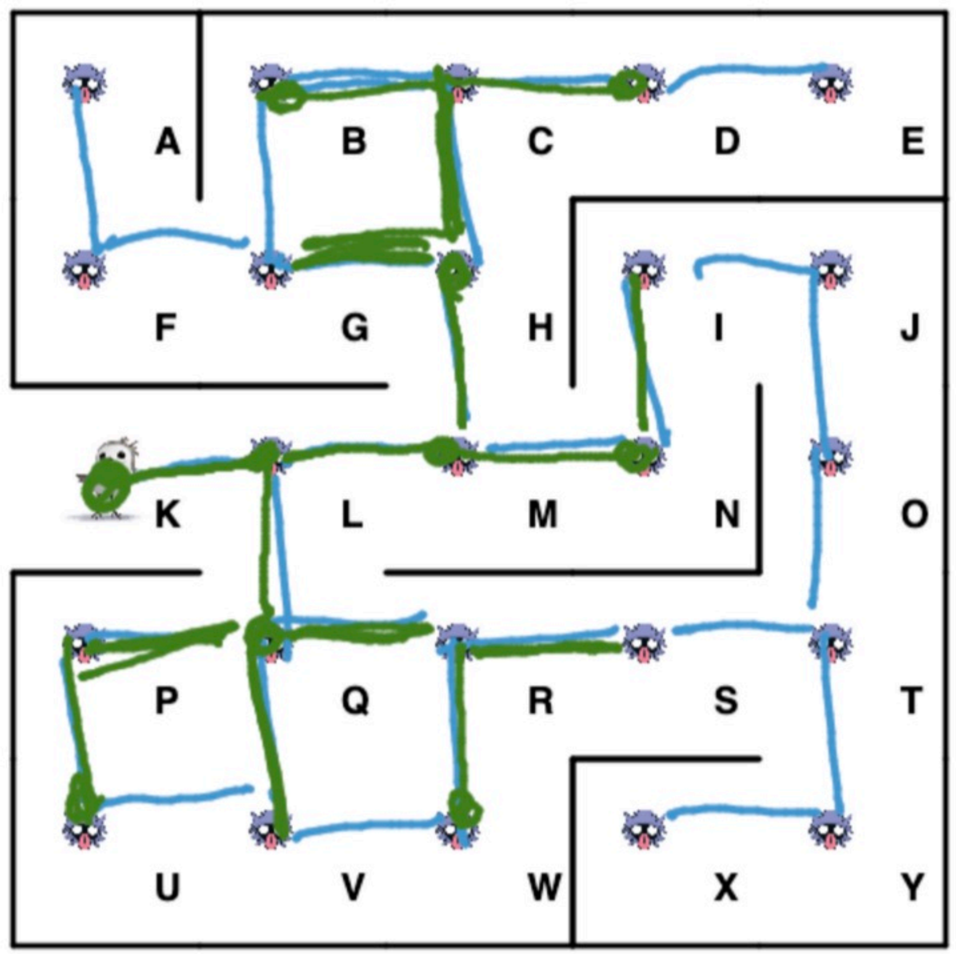


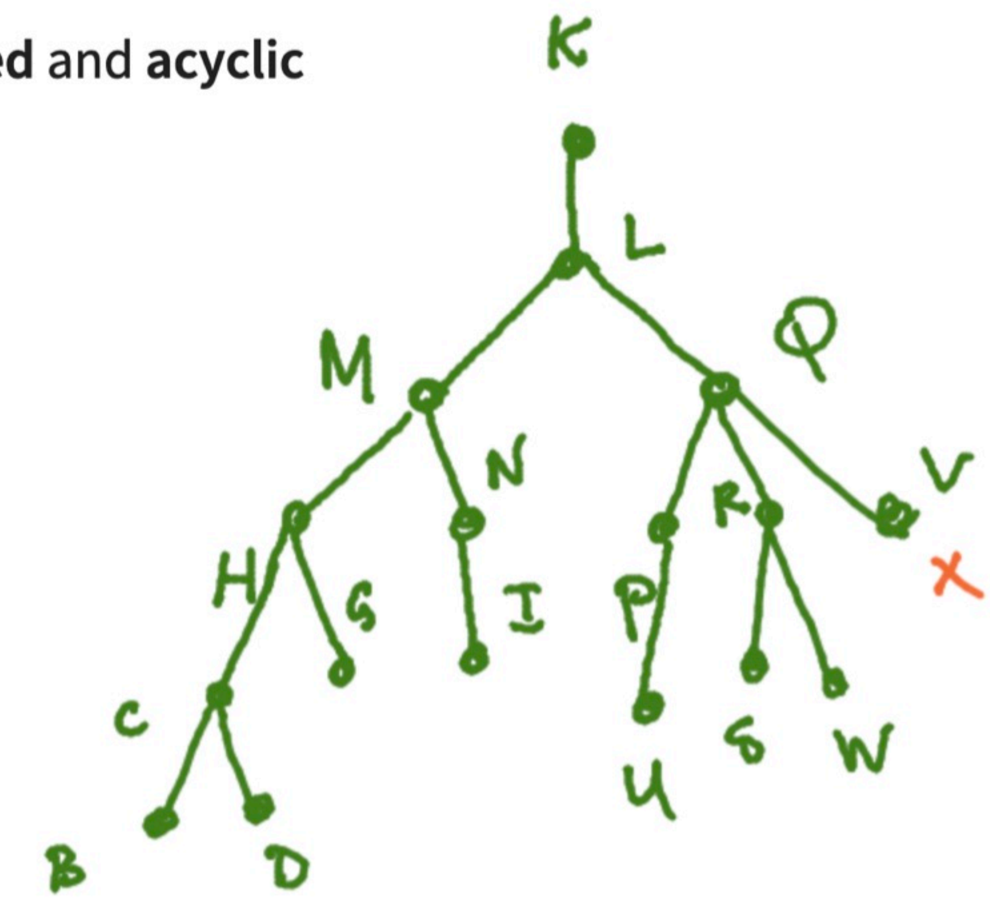
Solution: `[b, a, d, c, g, e, f]`

Breadth-First Search ("flooding").

K - L - M - Q - H - N - P - R - V - C - G - I - S - W - B - D ....

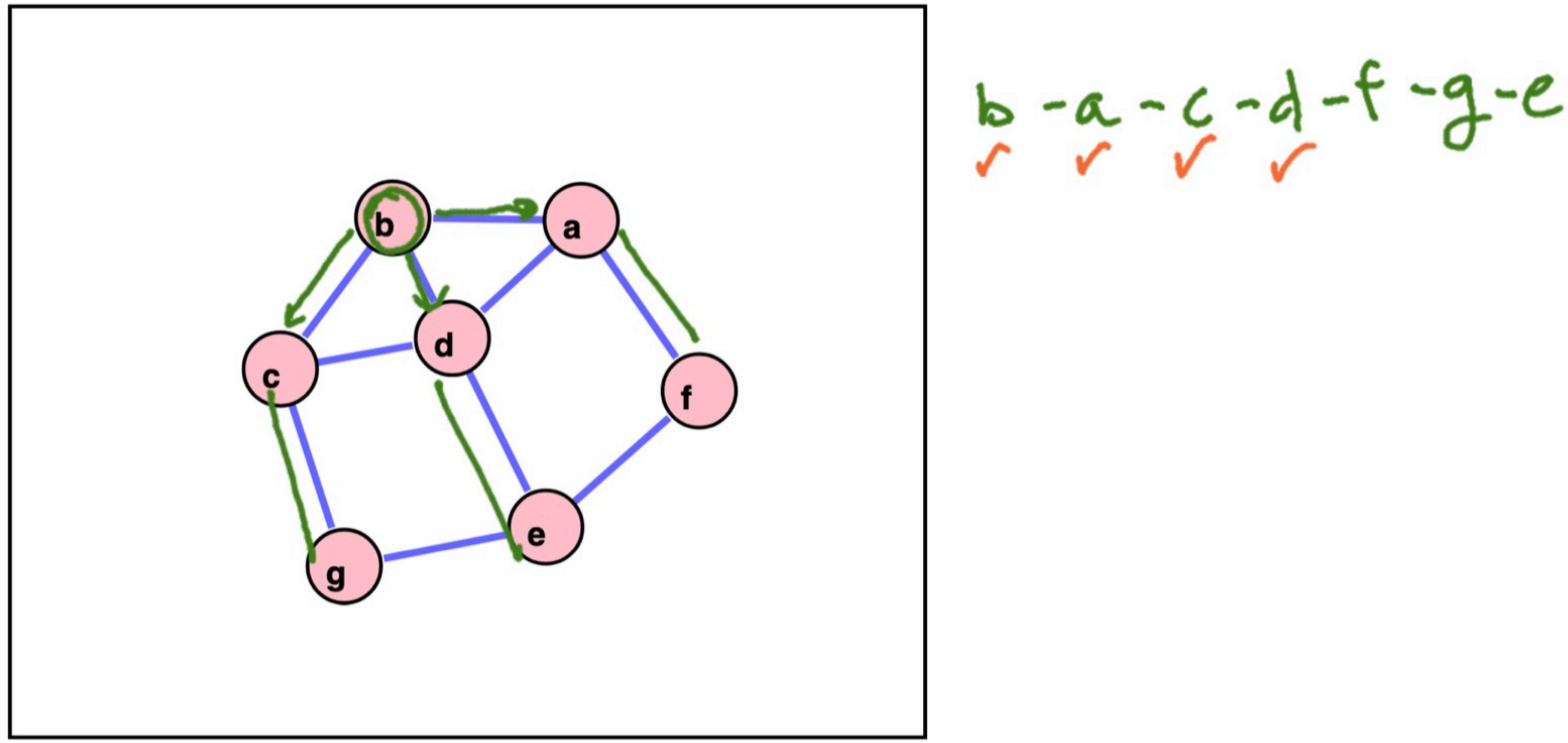
- **Main idea:** Visit neighbors one "level" at a time.
- Don't step into nodes we already visited.
- Resulting set of edges forms a **tree: connected** and **acyclic**





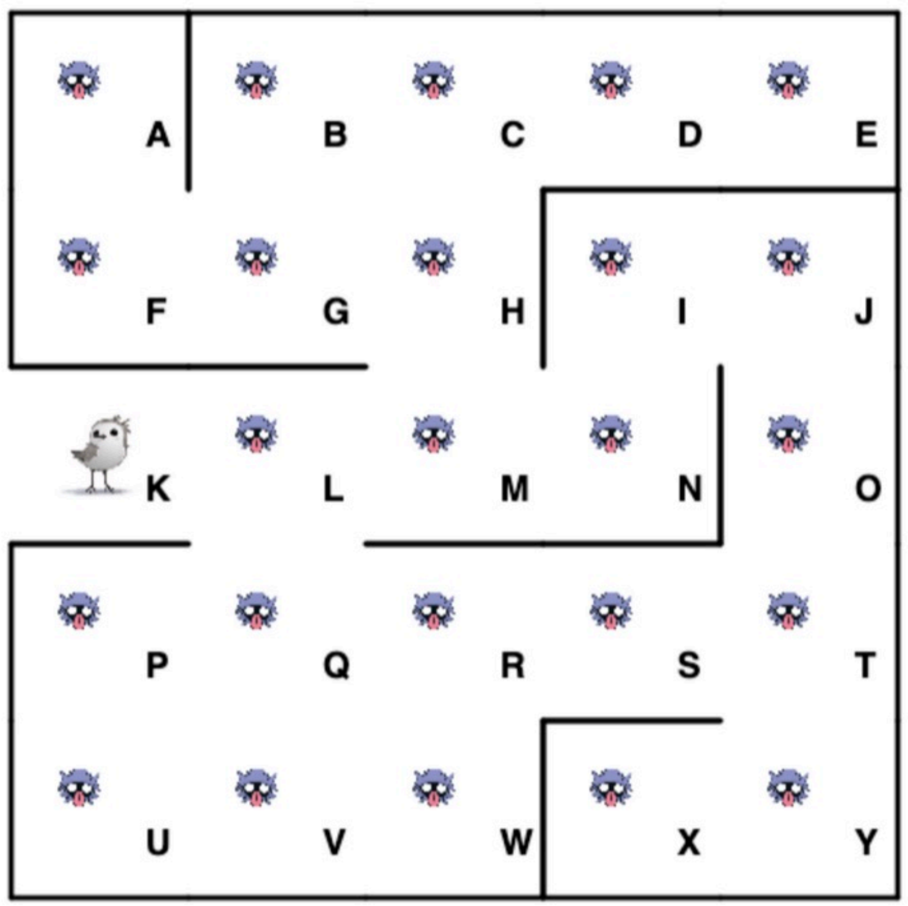
Exercise: visit all nodes using BFS, starting at b. List the order in which nodes are traversed.

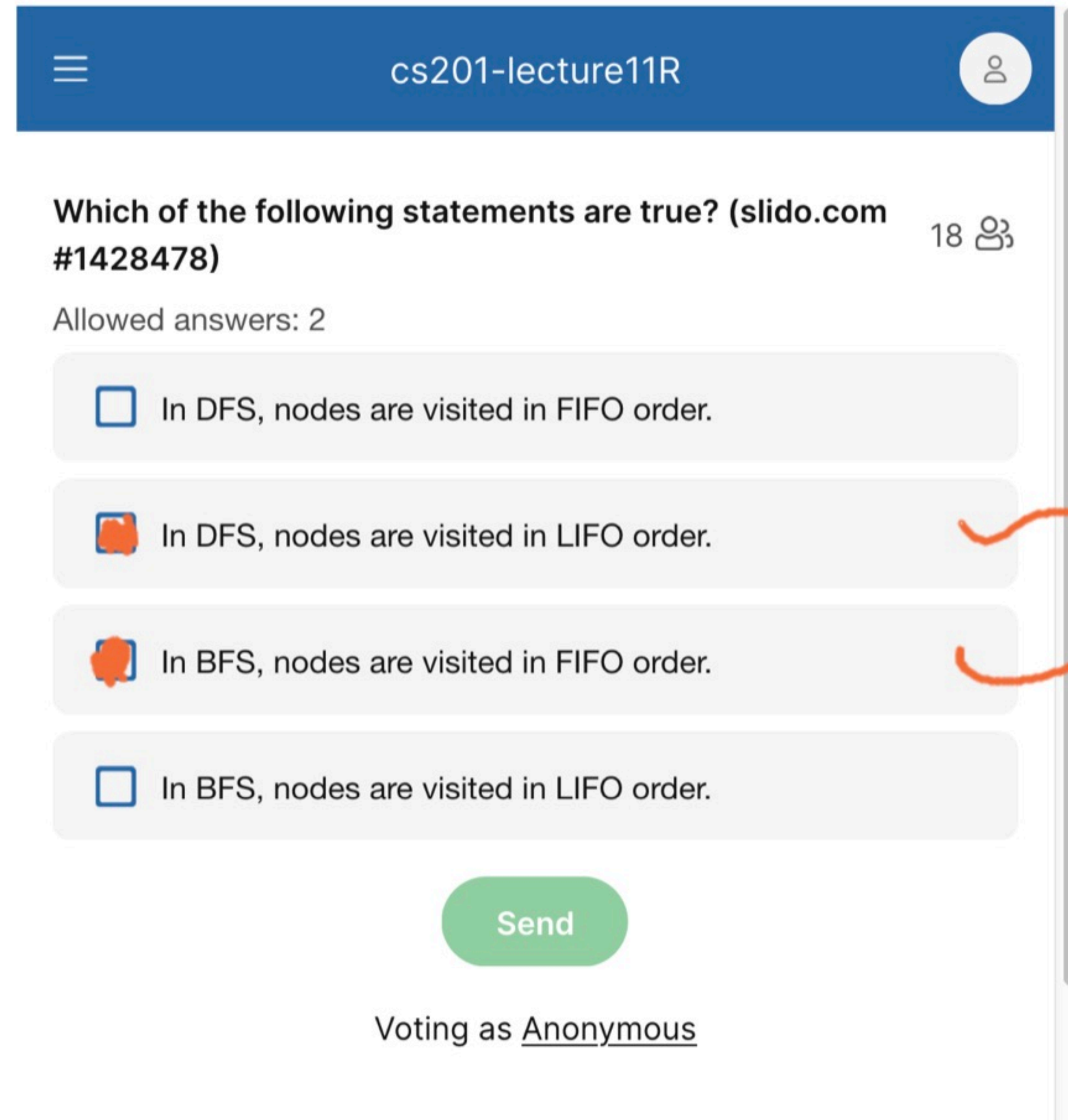
Assume we are using `TreeSet<Node>` to store adjacent nodes.



Solution: `[b, a, c, d, f, g, e]`

DFS steps into neighbors until we hit a wall. BFS steps into neighbors one level at a time.





stack

queue

Implementing DFS and BFS in Java.

Notice we are using TreeSet for adjacencies: neighboring nodes will be traversed in *order*.

```java
public class Graph<Node> {

  HashMap<Node, TreeSet<Node>> adj;

  public ArrayList<Node> dfs(Node root) {
    ArrayList<Node> order = new ArrayList<>();
    HashSet<Node> visited;
    dfsHelper(root, order, visited);
    return order;
  }

  private void dfsHelper(Node u,
                         ArrayList<Node> order,
                         HashSet<Node> visited) {
    visited.add(u);
    order.add(u);

    // TODO what lines could go here
    // to visit all the adjacent nodes of u?
  }
}
```

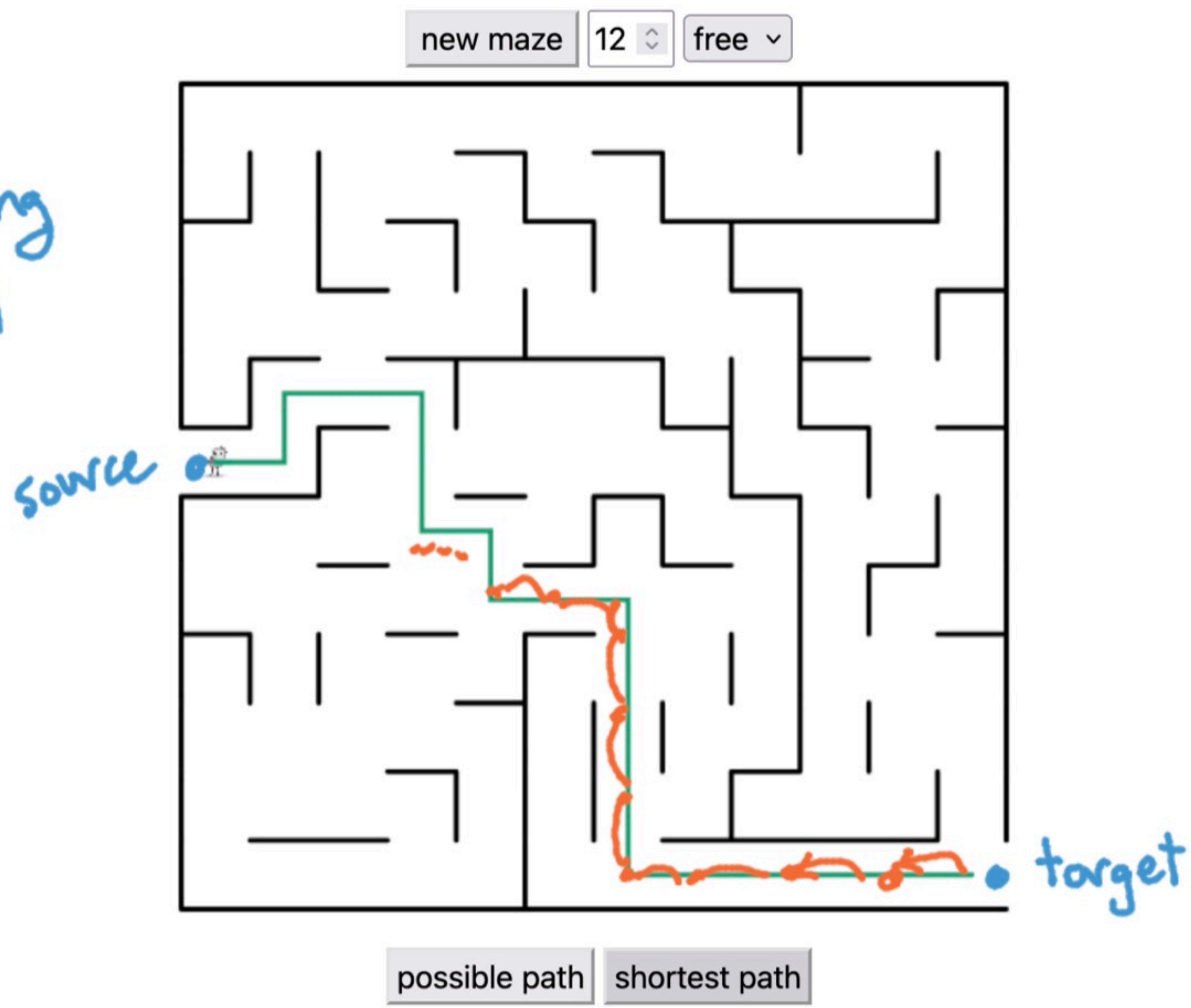
```java
public class Graph<Node> {

  HashMap<Node, TreeSet<Node>> adj;

  public ArrayList<Node> bfs(Node root) {
    ArrayDeque<Node> queue = new ArrayDeque<>();
    ArrayList<Node> order = new ArrayList<>();
    HashSet<Node> visited = new HashSet<>();

    queue.add(root);
    visited.add(root);
    while (!queue.isEmpty()) {
      Node u = queue.poll();
      order.add(u);

      // TODO what lines could go here
      // to visit all the adjacent nodes of u?
    }
    return order;
  }
```

```java
// possible implementation:
for (Node v : adj.get(u)) {
  if (!visited.contains(v)) {
    dfsHelper(v, order, visited);
  }
}
```

```java
// possible implementation:
for (Node v : adj.get(u)) {
  if (!visited.contains(v)) {
    visited.add(v);
    queue.add(v);
  }
}
```

Finding the shortest path between nodes (unweighted graphs).

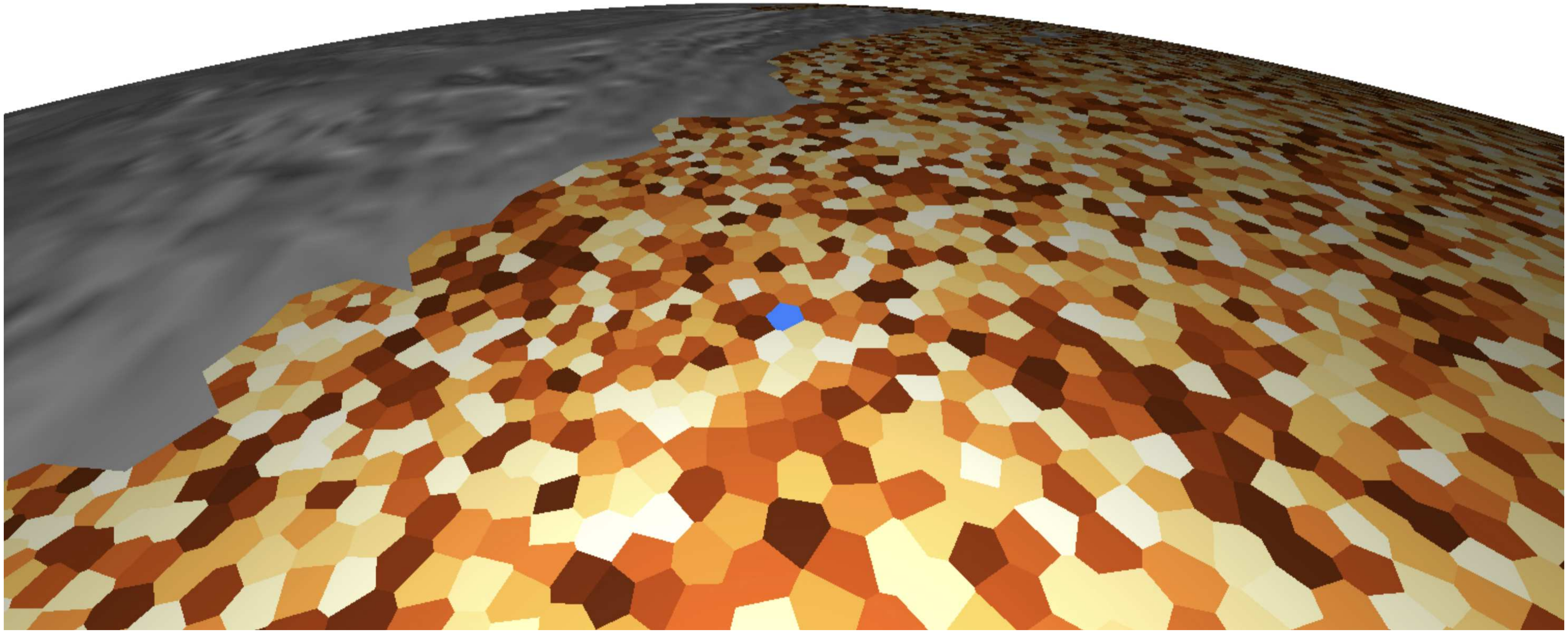
use BFS starting at source until we reach target



levels = length of shortest.

+ keep track of parent

Deciding when to use DFS or BFS.



Additional notes:

- Complete Exit Ticket 11R by end of today.
- **Homework 9** due date changed to Tuesday 12/3.