Middlebury

CSCI 201: Data Structures

Fall 2024

Lecture 5T: Recursion

Goals for today:

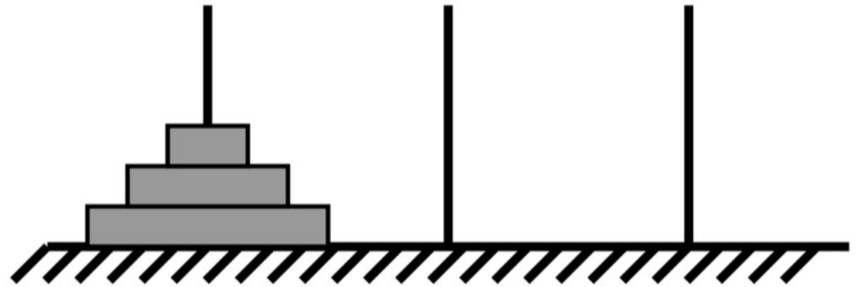
- Associate a call to a method with a **frame**.
- Use **recursion** to solve problems by building a solution from solutions to smaller problems.
- Ensure we always have a **base case** in our recursive solutions.
- Use **tail recursion** so that there are no pending operations after the recursive call.
- Practice some more with big-oh notation.

↳ + approaching the base case.

n > 3 7

n = 4 15

n = 5 31



how many "moves" to displace stack of n disks? (n = 3, here)

Rules: (also play here! https://www.mathsisfun.com/games/towerofhanoi.html)

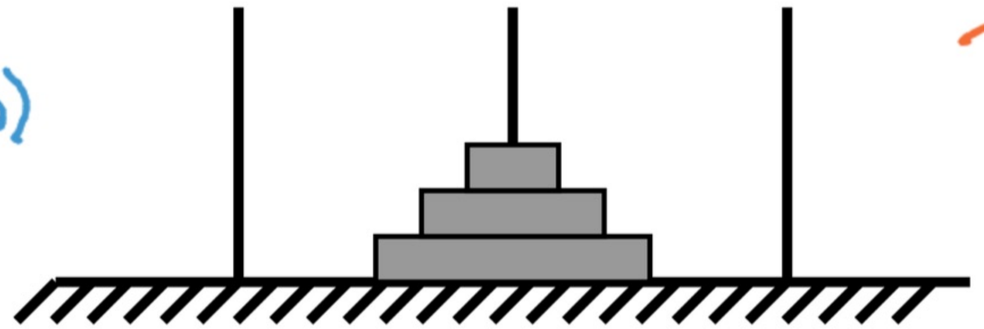
1. Move one disk at a time.
2. Every move displaces a disk from the top of one stack to the top of another (or empty rod).
3. No larger disk can ever be placed on top of a smaller one.

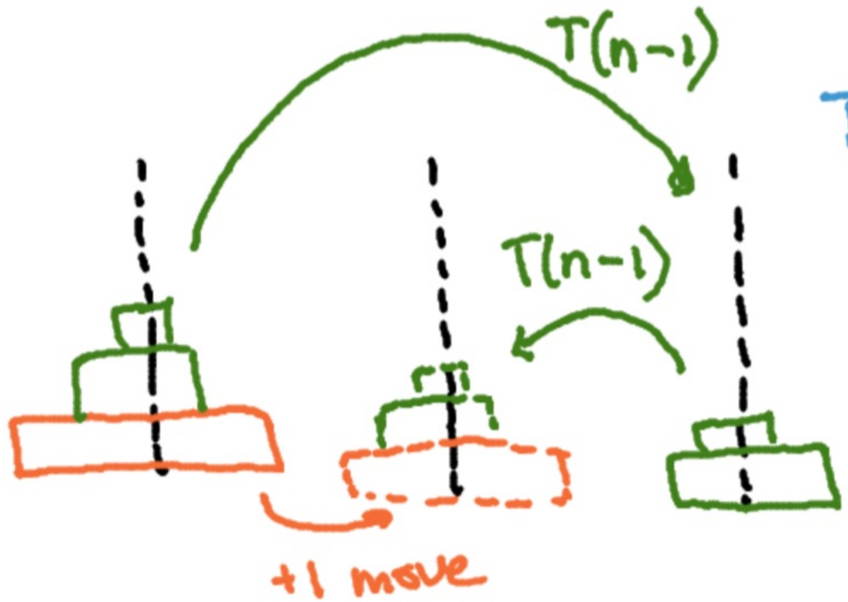
Counting the number of "moves" in the Tower of Hanoi problem.

think of "moves" like operations (last week)

$T(n)$

HHT || 7





recursive

$T(n) = 2T(n-1) + 1$

$T(1) = 1$ (base case)

Two ingredients for a recursive method:

- **Base case:** represents a problem you know how to solve.
- **Recursive case:** builds the solution to a problem from smaller subproblems.

Arithmetic series:

$$1 + 2 + 3 + \cdots + (n-1) + n$$

$= \frac{n(n+1)}{2}$

```java
public static int sum(int n) {
  if (n == 1) { // base case
    return 1;
  }
  // recursive case
  return n + sum(n - 1);
}
```

approach n == 1

Factorial:

$$1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$$

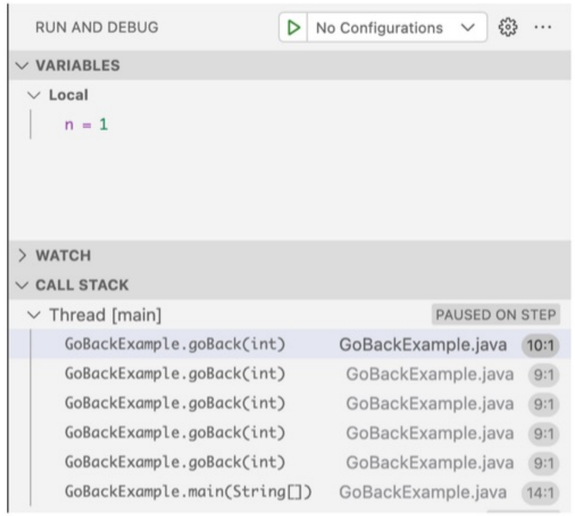
```java
public static int factorial(int n) {
  if (n < 2) { // base case
    return 1;
  }
  // recursive case
  return n * factorial(n - 1);
}
```

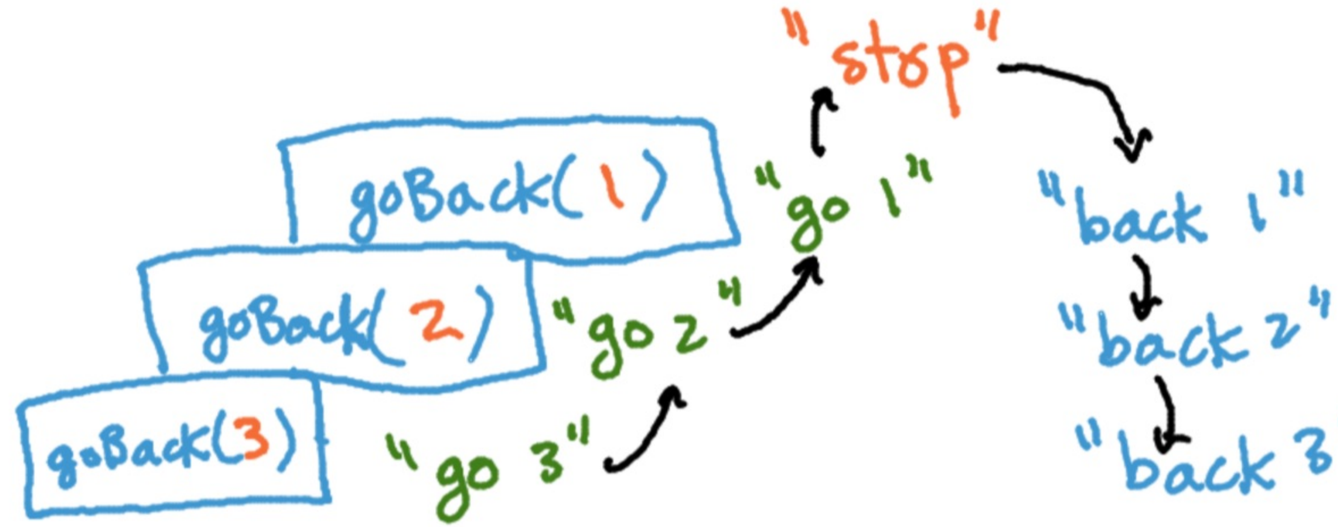
approach n < 2.

The recursive case needs to make progress towards the base case!

What does the `Call Stack` look like?

Open `GoBackExample.java` and add breakpoint on Line 6.






```java
public static void goBack(int n) {
  if (n == 0) {
    System.out.println("Stop");
  } else {
    System.out.println("Go " + n);
    goBack(n - 1);
    System.out.println("Back " + n);
  }
}
```

recursion (n.)

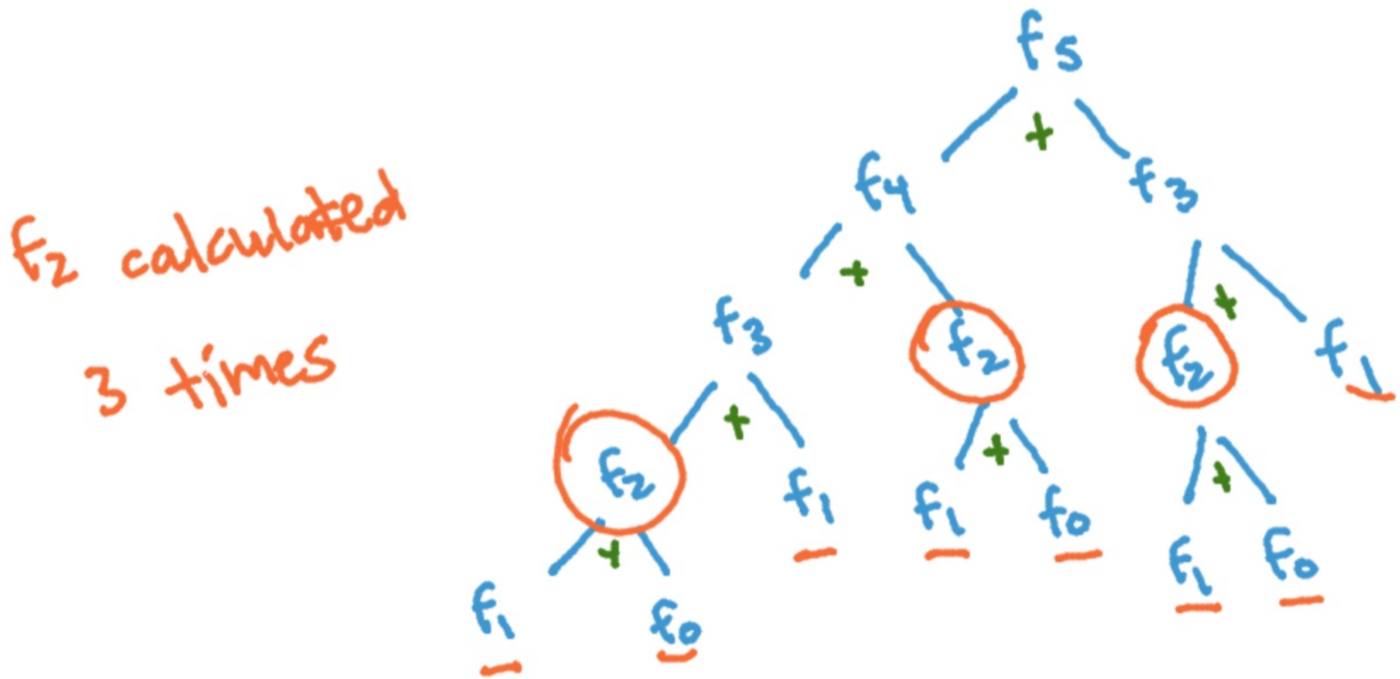
"return, backward movement," 1610s, from Latin *recursionem* (nominative *recursio*) "a running backward, return," noun of action from past-participle stem of *recurrere* "run back" (see **recur**).

Exercise: write a recursive function to compute Fibonacci numbers.

$$f(n) = f(n-1) + f(n-2), \quad \text{with } f(0) = 0,\ f(1) = 1.$$

```java
public static int fib(int n) {
  if (n < 2) {
    return n;
  }
  return fib(n - 1) + fib(n - 2);
}
```

Try $n = 40, n = 45, n = 48$. What do you notice?



* levels ≈ n
* branches = 2

$\approx 2^n$ calls to fib

$O(2^n)$

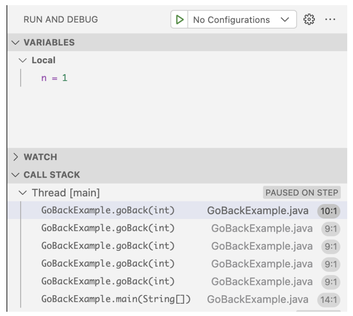
exponential

slow :(

The downsides of recursion.

- Performance might not be great.
- Recursion depth is limited by the stack size.

Go back to `GoBackExample.java` (remove breakpoints) and try $n = 100000$.

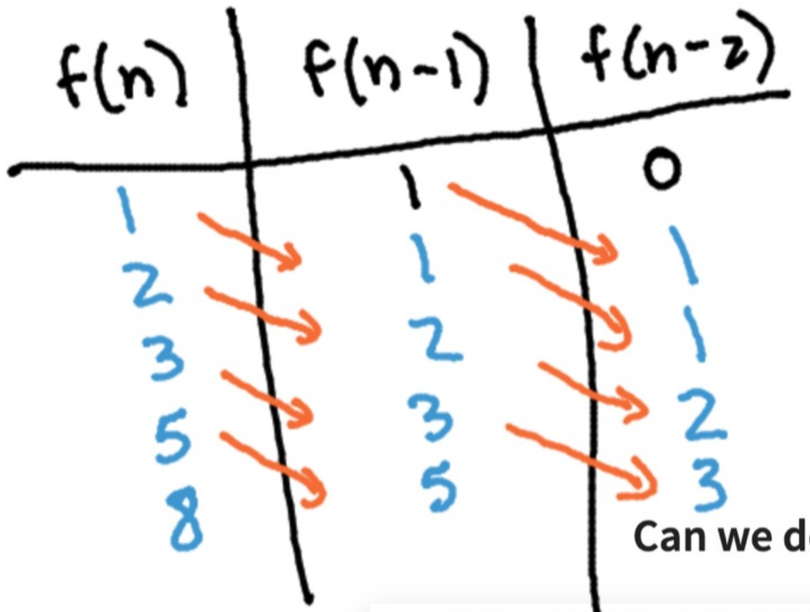



```java
public static void goBack(int n) {
  if (n == 0) {
    System.out.println("Stop");
  } else {
    System.out.println("Go " + n);
    goBack(n - 1);
    System.out.println("Back " + n);
  }
}
```

```
Go
Go
Go
Exception in thread "main" java.lang.StackOverflowError
        at java.base/java.io.FileOutputStream.write(FileOutputStream.java:349)
        at java.base/java.io.BufferedOutputStream.flushBuffer(BufferedOutputStream.java:81)
        at java.base/java.io.BufferedOutputStream.flush(BufferedOutputStream.java:142)
        at java.base/java.io.PrintStream.write(PrintStream.java:570)
        at java.base/sun.nio.cs.StreamEncoder.writeBytes(StreamEncoder.java:234)
        at java.base/sun.nio.cs.StreamEncoder.implFlushBuffer(StreamEncoder.java:313)
        at java.base/sun.nio.cs.StreamEncoder.flushBuffer(StreamEncoder.java:111)
        at java.base/java.io.OutputStreamWriter.flushBuffer(OutputStreamWriter.java:178)
        at java.base/java.io.PrintStream.writeln(PrintStream.java:723)
        at java.base/java.io.PrintStream.println(PrintStream.java:1028)
        at GoBackExample.goBack(GoBackExample.java:8)
        at GoBackExample.goBack(GoBackExample.java:9)
        at GoBackExample.goBack(GoBackExample.java:9)
```

Can we make our Fibonacci number calculation more efficient?

Let's try to write it as a `while`-loop instead.




```java
public static int fibWhile(int n) {
  int fnMinus2 = 0;
  int fnMinus1 = 1;
  int i = n;
  while (i > 1) {
    int fn = fnMinus1 + fnMinus2;
    fnMinus2 = fnMinus1;
    fnMinus1 = fn;
    i--;
  }
  return fnMinus1;
}
```

Can we do something similar using recursion?

```java
public static int fib(int n) {
  if (n < 2) return n;
  return fibHelper(n, 1, 0);
}

private static int fibHelper(int n, int fnMinus1, int fnMinus2) {
  if (n == 1) return fnMinus1;
  return fibHelper(n - 1,          ,          );
}
```

Can we make our *recursive* Fibonacci number calculation more efficient?

```
1  public static int fibWhile(int n) {
2    int fnMinus2 = 0;
3    int fnMinus1 = 1;
4    int i = n;
5    while (i > 1) {
6      int fn = fnMinus1 + fnMinus2;
7      fnMinus2 = fnMinus1;
8      fnMinus1 = fn;
9      i--;
10   }
11   return fnMinus1;
12 }
```

```
1 public static int fib(int n) {
2   if (n < 2) return n;
3   return fibHelper(n, 1, 0);
4 }
5
6 private static int fibHelper(int n, int fnMinus1, int fnMinus2) {
7   if (n == 1) return fnMinus1;
8   return fibHelper(n - 1,          ,          );
9 }
```

1

fnMinus1);

fnMinus1 + fnMinus2

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What should the recursive call to fibHelper look like on Line 8? (slido.com #3769190)

33

- ○ fibHelper(n - 1, fnMinus1, fnMinus1 + fnMinus2);
- ○ fibHelper(n - 1, fnMinus2, fnMinus1 + fnMinus2);
- fibHelper(n - 1, fnMinus1 + fnMinus2, fnMinus1);

Send

Voting as Anonymous

Two things going on here:

- Computing any given Fibonacci number *once*.
- **Tail-Recursive:** the recursive call is the **last** thing done in the method.
  - **Strategy to develop:** think about how to write it as a `while`-loop first.

helper

```
1 public static int fib(int n) {
2   if (n < 2) return n;
3   return fibHelper(n, 1, 0);
4 }
5
6 public static int fibHelper(int n, int fnMinus1, int fnMinus2) {
7   if (n == 1) return fnMinus1;
8   return fibHelper(n - 1, fnMinus1 + fnMinus2, fnMinus1);
9 }
```

How many times is `fibHelper` called?

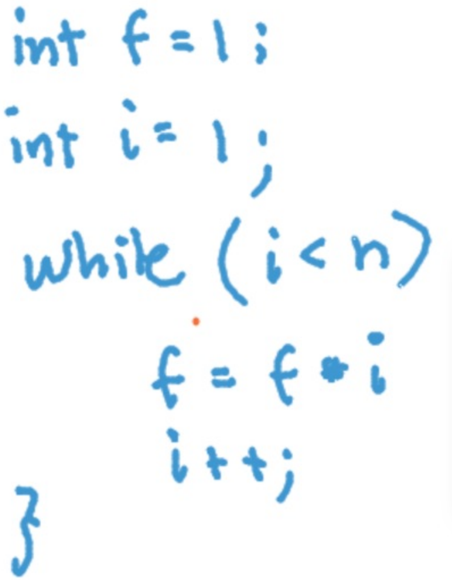
Same as while loop

O(n)

why tail recursion? Some compilers will detect it's the last thing called and will optimize out the creation of a new stack frame.

Exercise: rewrite `factorial` to be tail-recursive.

```java
public static int factorial(int n) {
  if (n < 2) { // base case
    return n;
  }
  // recursive case
  return n * factorial(n - 1);
}
```




```java
public static int factorialTail(int n) {
  return factorialTailHelper(n, 1);
}

private static int factorialTailHelper(int n, int result) {
  if (n == 1) return result;
  return factorialHelper(n - 1, n * result);
}
```

f

f

In this case, we still have $\mathcal{O}(n)$ multiplications ($*$), but a compiler might perform *tail call optimization* (TCO), eliminating the need for a new stack frame (just not in Java).

See you on Thursday!

- Lab 4 (Bucket Sort) due tonight at 11:59pm.
- Start Homework 4!
    1. Implement Radix Sort.
    2. Derive number of `=` for `DIYList` `add` method with a different growth technique.
- Reminder that Noah (go/noah) and Smith (go/smith) have office hours throughout the week and the 201 Course Assistants have drop-in hours in the late afternoons/evenings (go/cshelp).
- Submit exit ticket 5T today.