Middlebury

CSCI 200: Math Foundations of Computing

Spring 2026

Lecture 5M: Deduction

Try to form a conclusion from these statements:

If I wake up early, then I will drink coffee.
If I drink coffee, then I will be full of energy.
I woke up early. Therefore, ________________. I will be full of energy.

Can we structure this deduction?
How can we check our conclusion?

Goals for today

- Identify **variables**, **premises** and **conclusions**.
- Use a **truth table** to check your deductions.
- Deduce new truths from existing statements using **rules of inference**.

If you already have a conclusion, check it with a truth table.

a *If I wake up early, then I will drink coffee.* $\underbrace{\text{If I wake up early}}_{p}$, then $\underbrace{\text{I will drink coffee}}_{q}$ — r

b *If I drink coffee, then I will be full of energy.*

c *I woke up early. Therefore, ______________.* I will be full of energy

2 $\underbrace{\text{I woke up early}}_{p}$

a $p \to q$

b $q \to r$

c p

when are premises true?

truth table:

p	q	r	$p \to q$	$q \to r$	p	r
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	F	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	F	T
F	T	F	T	F	F	F
F	F	T	T	T	F	T
F	F	F	T	T	F	F

conclusion:

r

Four rules of inference to chain premises when forming new truths. (4 for us).

(modus ponens)

rule #1: variables p, q

premises:

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore\ q \end{array}$$

"therefore" → $\therefore$

vars		premises		concl.
p	q	p	$p \to q$	q
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

rule #2:

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore\ p \to r \end{array}$$

hypothetical syllogism

rule #3:

$$\begin{array}{l} \neg p \to \neg q \\ \hline \therefore\ q \to p \end{array}$$

contrapositive

rule #4:

$$\begin{array}{l} \neg q \\ p \to q \\ \hline \therefore\ \neg p \end{array}$$

modus tollens

Example: use rules of inference to form a conclusion.

If I wake up early, then I will drink coffee.
If I drink coffee, then I will be full of energy.
I woke up early. Therefore, ________________.

a) $p \to q$
b) $q \to r$
c) p

step	statement (true)	notes
1	$p \to r$	rule #2 with a and b
2	r	rule #1 with step 1 and c
	$\therefore r$	

Rule 1:

$$\begin{array}{r} p \\ p \implies q \\ \hline \therefore q \end{array}$$

Rule 2:

$$\begin{array}{r} p \implies q \\ q \implies r \\ \hline \therefore p \implies r \end{array}$$

Rule 3:

$$\begin{array}{r} \neg p \implies \neg q \\ \hline \therefore q \implies p \end{array}$$

Rule 4:

$$\begin{array}{r} \neg q \\ p \implies q \\ \hline \therefore \neg p \end{array}$$

Different ways to say $p \implies q$.

- p implies q.
- If p, then q.
- If p, q.
- p only if q.
- q whenever p.
- q follows from p.
- q if p.
- q when p.
- q is necessary for p.
- A necessary condition for p is q.
- p is sufficient for q.
- A sufficient condition for q is p.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

car starts

there is fuel in the tank

Example: Will I finish my homework?

I go skiing only if there is fresh snow.
If I don't go skiing, then I'll go to office hours.
If I go to office hours, then I will finish my homework.
There isn't fresh snow today but it's warmer than yesterday.

a $s \to f$
b $\neg s \to o$
c $o \to h$
d $\neg f \cancel{\land w}$

- f: There is fresh snow.
- s: I will go skiing.
- o: I will go to office hours.
- h: I will finish my homework.

w: warmer than yesterday

step	statement (true)	notes
1.	$\neg f$	simplify d
2.	$\neg f \to \neg s$	rule 3 with a
3.	$\neg s$	rule 1, step 1, 2
4.	o	rule 1, step 3 + b
	$\therefore h$	rule 1, step 4 + c

Rule 1:

$$\begin{array}{r} p \\ p \implies q \\ \hline \therefore q \end{array}$$

Rule 2:

$$\begin{array}{r} p \implies q \\ q \implies r \\ \hline \therefore p \implies r \end{array}$$

Rule 3:

$$\begin{array}{r} \neg p \implies \neg q \\ \hline \therefore q \implies p \end{array}$$

Rule 4:

$$\begin{array}{r} \neg q \\ p \implies q \\ \hline \therefore \neg p \end{array}$$

A puzzle by Lewis Carroll.

All hummingbirds are richly colored.
No large birds live on honey.
Birds that do not live on honey are dull in color.
Hummingbirds are ___________
Hint: convert existential quantifier to universal quantifier.

Rule 1:

$$\frac{\begin{array}{r} p \\ p \implies q \end{array}}{\therefore q}$$

Rule 2:

$$\frac{\begin{array}{r} p \implies q \\ q \implies r \end{array}}{\therefore p \implies r}$$

Rule 3:

$$\frac{\neg p \implies \neg q}{\therefore q \implies p}$$

Rule 4:

$$\frac{\begin{array}{r} \neg q \\ p \implies q \end{array}}{\therefore \neg p}$$