Middlebury

CSCI 200: Math Foundations of Computing

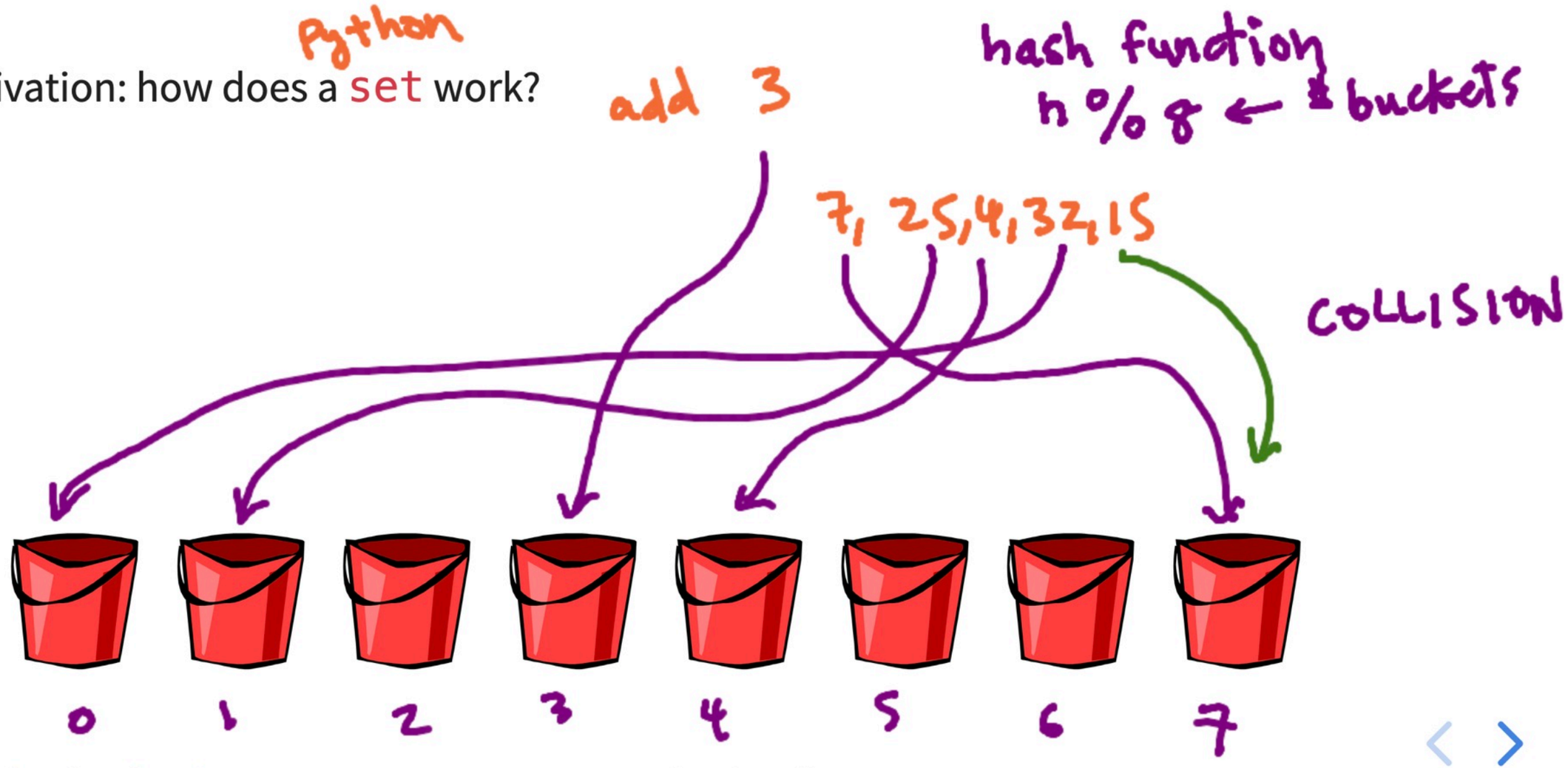
Spring 2026

Lecture 3M: Functions

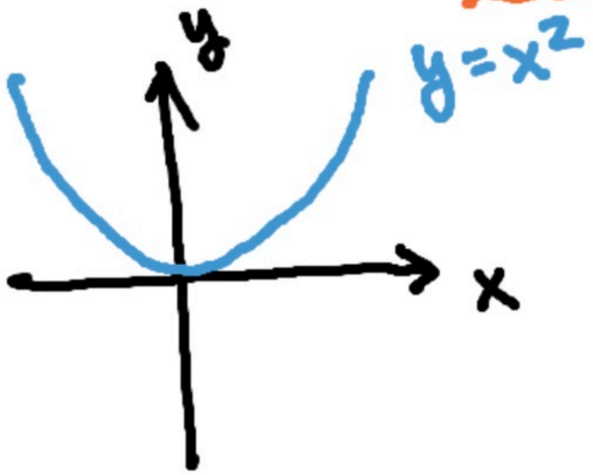
Goals for today:

1. Identity whether a map between two sets is a **function**.
2. Classify a given function as either **surjective**, **injective** or **bijective**.
3. Compare the sizes of infinite sets.

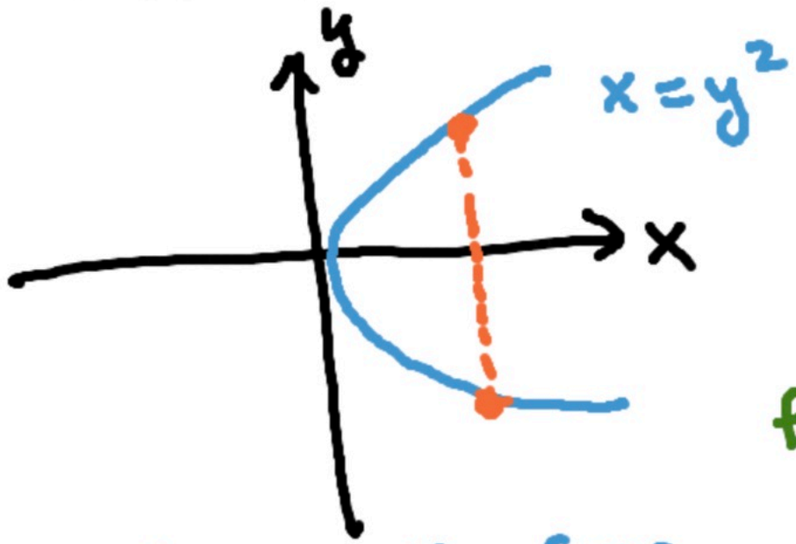
Motivation: how does a `set` work?



A function is a map from sets $A \to B$ such that *every* element of A maps to a *unique* element of B.



function from x→y



not a function from x→y

codomain

$f : A \to B$

function — domain

$a \in A$

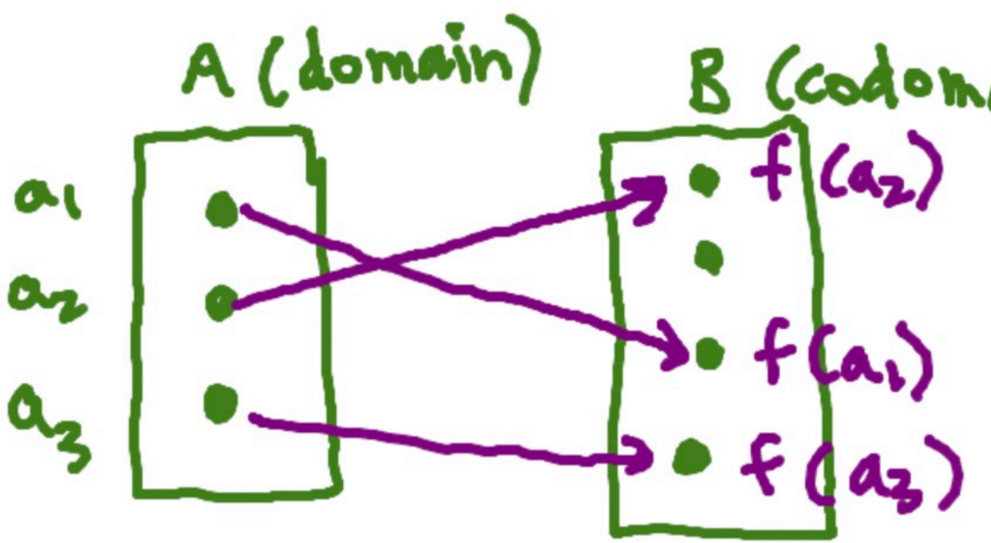
$b \in B$

$f(a) = b$

↳ b is the image of a

$f^{-1}(b) = a$

↳ a is the preimage of b



range: set of all images

here, $\{f(a_2), f(a_1), f(a_3)\}$

Question 1: Identify the domain, codomain and range of the following functions.

1. $f: \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 2n$.

codomain

domain

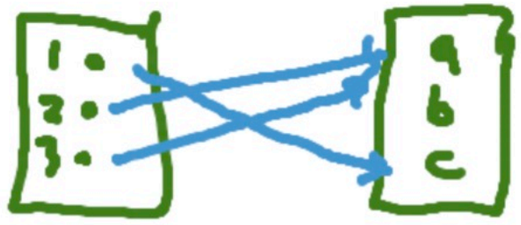
range: all even integers

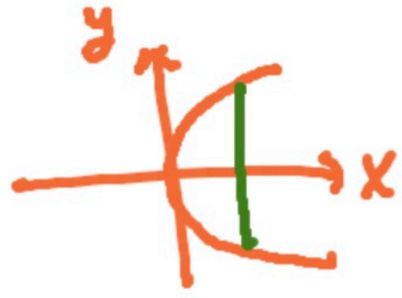
2. $g: \{1, 2, 3\} \to \{a, b, c\}$ defined by $g(1) = c, g(2) = a$ and $g(3) = a$.

domain

codomain

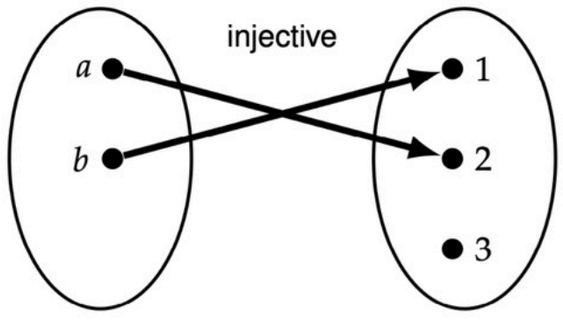
range: $\{a, c\}$



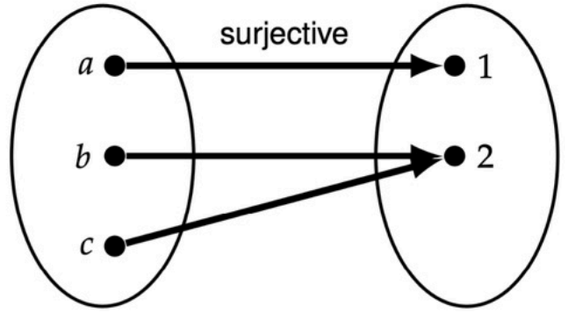


Properties of functions: injective, surjective, bijective.

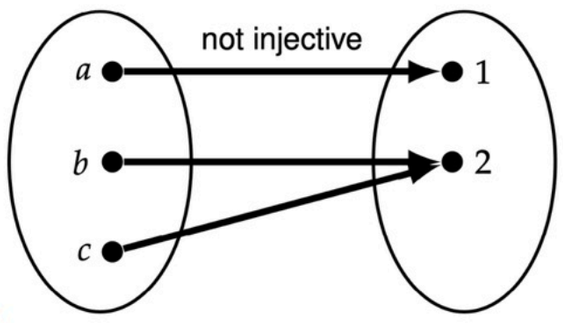
Injective: *one-to-one*



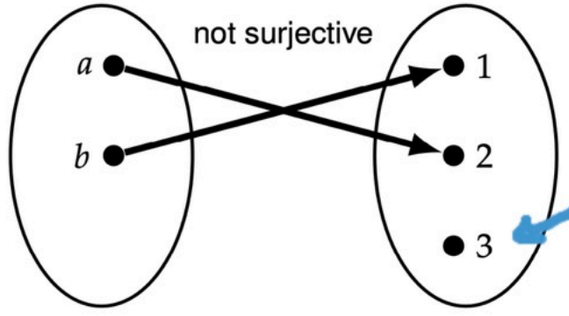
Surjective: *onto*



range is the same as codomain

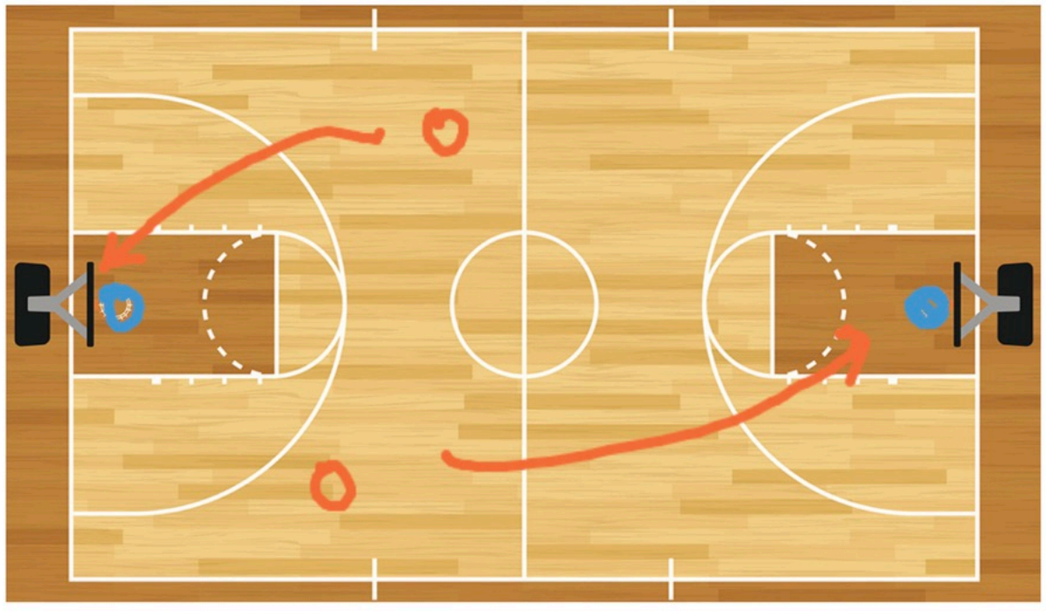


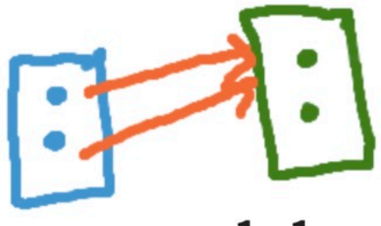
2 different elements map to same element in codomain.



left out

Consider the following basketball situations. Let the domain be the current players and let the codomain (always) be the two nets. Assume everyone scores at some point in the game.





Function? Injective? Surjective? Bijective?

1. 1-on-1 half-court game. not injective, not surjective
2. 5-on-5 full-court game. not injective, surjective ✓
3. 1-on-1 full-court game. injective, surjective.
4. Someone scored on their own net after scoring on the opposing team's net. not a function

function ✓

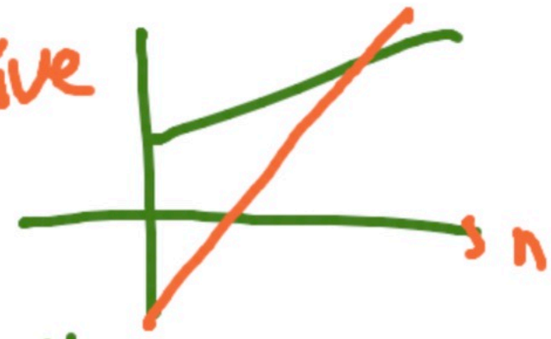
Question 2: Determine if the following functions are injective, surjective or bijective.

1. $f: \mathbb{N} \to \mathbb{N}$ where $f(n) = n + 5$.

injective
not surjective $\{1, 2, 3, 4, 5\}$

2. $f: \mathbb{Z} \to \mathbb{Z}$ where $f(n) = n + 5$.

injective.
surjective
] bijective



3. $f: \mathbb{Z} \to \mathbb{Z}$ where $f(n) = 3n - 4$.

injective

can I get $f(n) = 0 = 3n - 4$
not surjective $n = \frac{4}{3} \notin \mathbb{Z}$

4. $f: \mathbb{Z} \to \mathbb{Z}$ where

n	f(n)
0	0
1	1
2	1
3	2
4	2

not injective
surjective

$$f(n) = \begin{cases} n/2 & n \text{ is even} \\ (n+1)/2 & n \text{ is odd.} \end{cases}$$

Question 3: how many surjective maps are there from n boolean variables to $\{$`True`, `False`$\}$? Discuss and then vote.

A. 2^n

B. $2n$

C. $2^n - 2$

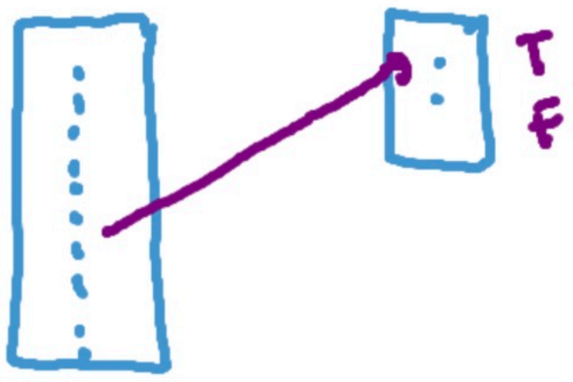
D. $2n - 2$

E. $2^n - 2n$

think about a truth table

2^n rows in truth table

all true or all false are not surjective



$2 \times 2 \times \cdots \times 2 = 2^n$ options total.

Is $|\mathbb{R}| = |\mathbb{N}|$?

A set S is *countably infinite* if there is a **bijection** from S to $\mathbb{N}$.
Example: $f: \mathbb{N} \to \mathbb{E}$ (even numbers) where $f(n) = 2n$.

What about $S = \{x \in \mathbb{Z} : x > 10\}$? Is $|S| = |\mathbb{N}|$?

However, we cannot create a bijection from $\mathbb{R} \to \mathbb{N}$.

Main argument (and a proof by contradiction) uses *diagonalization*.

↦ natural numbers

↗ f is bijective

yes let $f : \mathbb{N} \to S$ with $f(n) = n + 11$

or, let $g : S \to \mathbb{N}$ with $g(n) = n - 10$

↳ g is bijective

S is countably infinite

Is $|\mathbb{R}| = |\mathbb{N}|$?

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yes let $f: \mathbb{N} \to S$ with $f(n) = n + 10$

or, let $g: S \to \mathbb{N}$ with $g(n) = n - 10$

g is bijective

S is countably infinite

However, we cannot create a bijection from $\mathbb{R} \to \mathbb{N}$.

Main argument (and a proof by contradiction) uses *diagonalization*.

↦ natural numbers

f is bijective