reminders:

- pset 2 due on Fri. (quiz 2)
- CS talks Fri. 12:20 pm

Middlebury

CSCI 200: Math Foundations of Computing

Spring 2026

Lecture 2W: Quantifiers

Warmup: convert this word description to a set using set-builder notation.

$V_i =$ *the set of all points in the plane ($\mathbb{R}^2$) which are closer to point i than to any other point in a list of n points.*

domain.

$$d(x,y,a,b) = \sqrt{(x-a)^2+(y-b)^2}$$

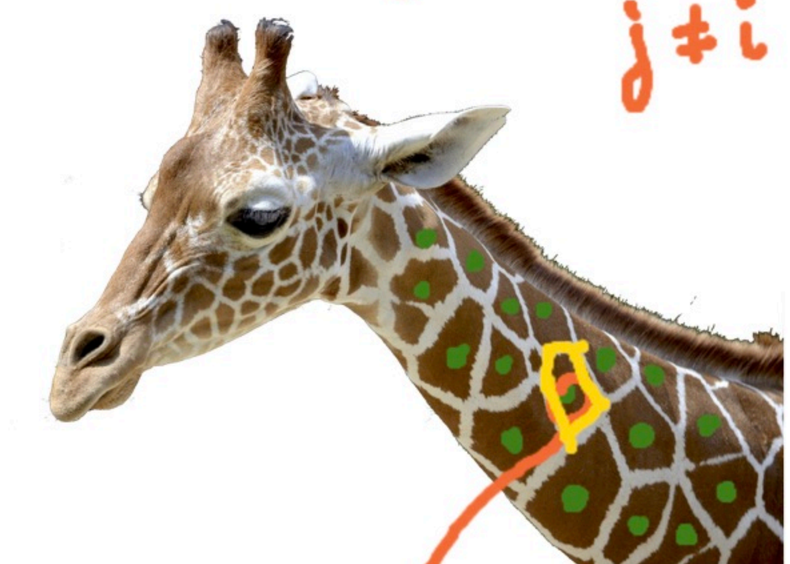
$$V_i = \{(x,y)\in\mathbb{R}^2 \mid d(x,y,x_i,y_i) < d(x,y,x_j,y_j)\ \forall j \neq i\}$$

Voronoi cell.

$j = 1, 2, \ldots, n$

$j \neq i$

universal quantifier



point i

(x_i, y_i)

Objectives for today:

- use the **universal** and **existential** quantifiers to quantify a predicate,
- negate quantified expressions,
- translate English statements to math.

Quantifiers can be used to quantify the values of predicates.

$p(x): x > 0$

universal quantifier

"for all"

comma

$$\underbrace{\forall x \in \mathbb{N},\ \underbrace{x > 0}_{\text{predicate}}}_{\text{statement (true)}}$$

domain

there exists

such that

existential quantifier

$$\underbrace{\exists x \in \mathbb{N} : \underbrace{x < 0}_{\text{predicate}}}_{\text{statement (false)}}$$

domain

Example 1: translate *Some birds can fly.*

Let A be the set of all animals.
Let $b(x)$ be the predicate that x is a bird.
Let $f(x)$ be the predicate that x can fly.

$\exists x \in A :$ $\boxed{① \ b(x) \wedge f(x)}$? ✓

② $b(x) \rightarrow f(y)$?

x = penguin
b(x): true
f(x): false
$T \rightarrow F$ is F

x = elephant
b(x): false
f(x): false
$F \rightarrow F$ is T
(empty)

Negating quantified expressions.

translate this.

S: students

Every Middlebury student lives on campus.

negation: NOT every Middlebury student lives on campus

$p(x)$: x lives on campus $\forall x \in S,\ p(x)$

translate → there is some student who does not live on campus

$\exists x \in S : \neg p(x)$

same as

$\neg(\forall x \in S,\ p(x))$

$\neg(\forall x \in S,\ p(x)) \equiv \exists x \in S : \neg p(x)$

$\neg(\exists x \in S : p(x)) \equiv \forall x \in S,\ \neg p(x)$

Some tips!

$\forall x, y \in S,\ p(x,y)$

- Combine multiple elements of same type with a quantifier:
- Any non-quantifier variables should be inputs to a predicate,
- When a variable is quantified, rewrite your predicate in terms of remaining variables:

  $\exists y \in S : p(x,y)$ rewrite as $q(x)$
- All variables in a statement should be quantified.

Example 2: Which of the following statements are true? Discuss and then vote.

Let S be the set of all people. Let $p(x, y)$ be the predicate that x is a parent of y.

A. $\forall x \in S, \exists y \in S\colon\ p(x, y)$ × everyone has a child

$\boxed{\text{B.}}$ $\forall x \in S, \exists y \in S\colon\ p(y, x)$ ✓ every person has a parent

C. $\exists x \in S\colon \forall y \in S,\ p(y, x)$ ×

A B C D E ✋ ❤️ 👍 👎 😳 🤔 😊 🐦 🐢 🐍

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Example 3: Translate the following to math.

Let S be the set of people in the class.
Let $F(x, y)$ mean that person x considers person y to be their friend ($x \neq y$).

1. Proposition p states that there is some super likable person in the class that everyone considers their friend.

$$p = \exists x \in S : \forall y \in S, F(y, x)$$

2. Proposition q states that everyone in the class has at least one person they consider to be their friend.

$$q = \forall x \in S, \exists y \in S : F(x, y)$$

3. Proposition r states that there is a mutual friendship in the class.

$$r = \exists x, y \in S : F(x, y) \wedge F(y, x)$$

4. Predicate $b(x, y)$ states that everyone who considers person x to be their friend also considers person y to be their friend.

$$b(x, y) = \forall z \in S, F(z, x) \to F(z, y)$$