Middlebury

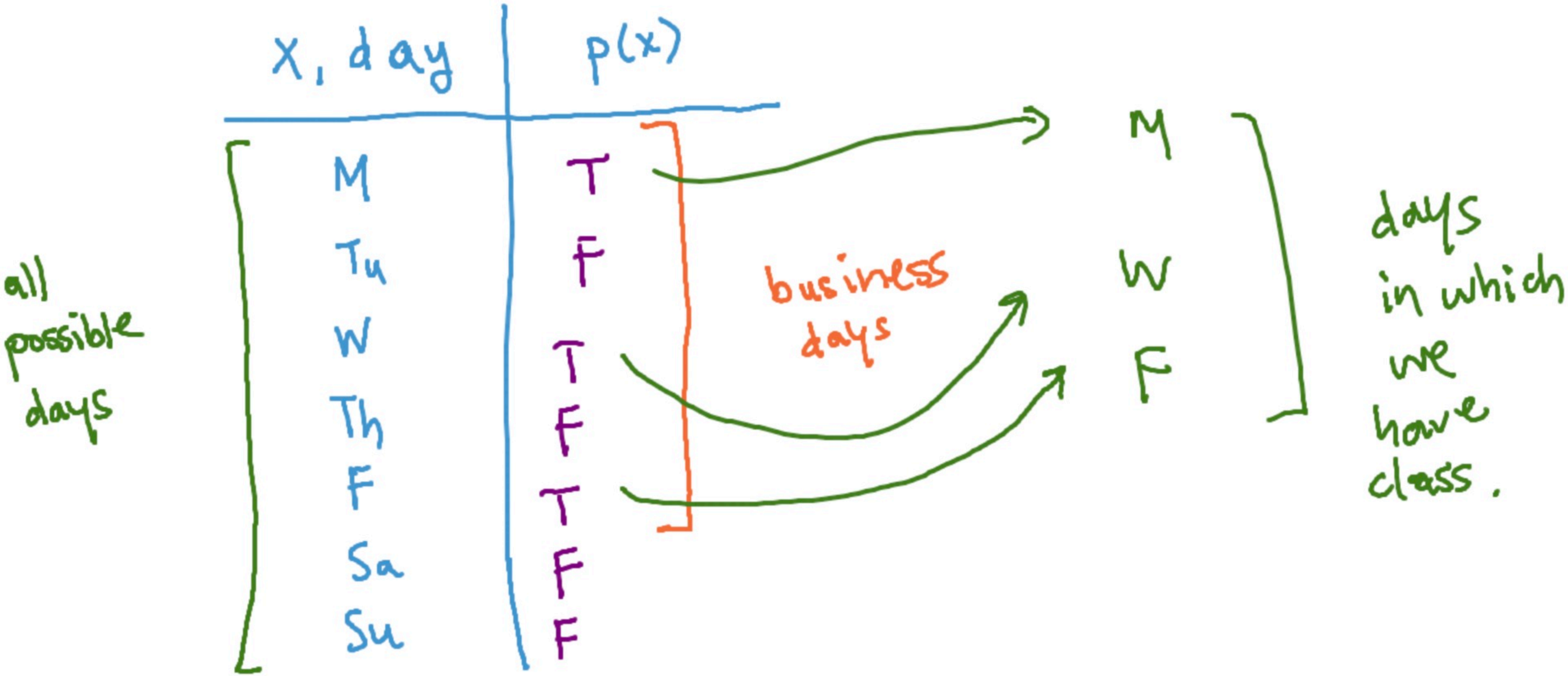
CSCI 200: Math Foundations of Computing

Spring 2026

Lecture 2M: Sets

Recall predicates (propositions that depend on variables).

Example: $p(x) =$ *We have class on* x.



Goals for today:

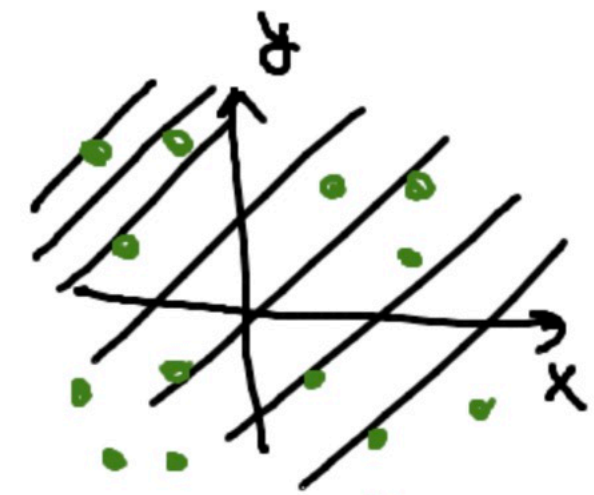
- Build sets using both **roster** and **set-builder** notation.
- Perform **operations** on sets and **subsets**.
- Use some famous sets: $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{N}$.

unordered

A *set* is a bunch of objects, called *elements*.

curly braces

- **set of animals:** { cat, dog, penguin, elephant, ... }
- **set of even numbers:** { 2, 4, 6, 8, 10, 12, ... }
- **set of points on the plane:** { (0,0), (1,0), (0,1), (1,1), ... }

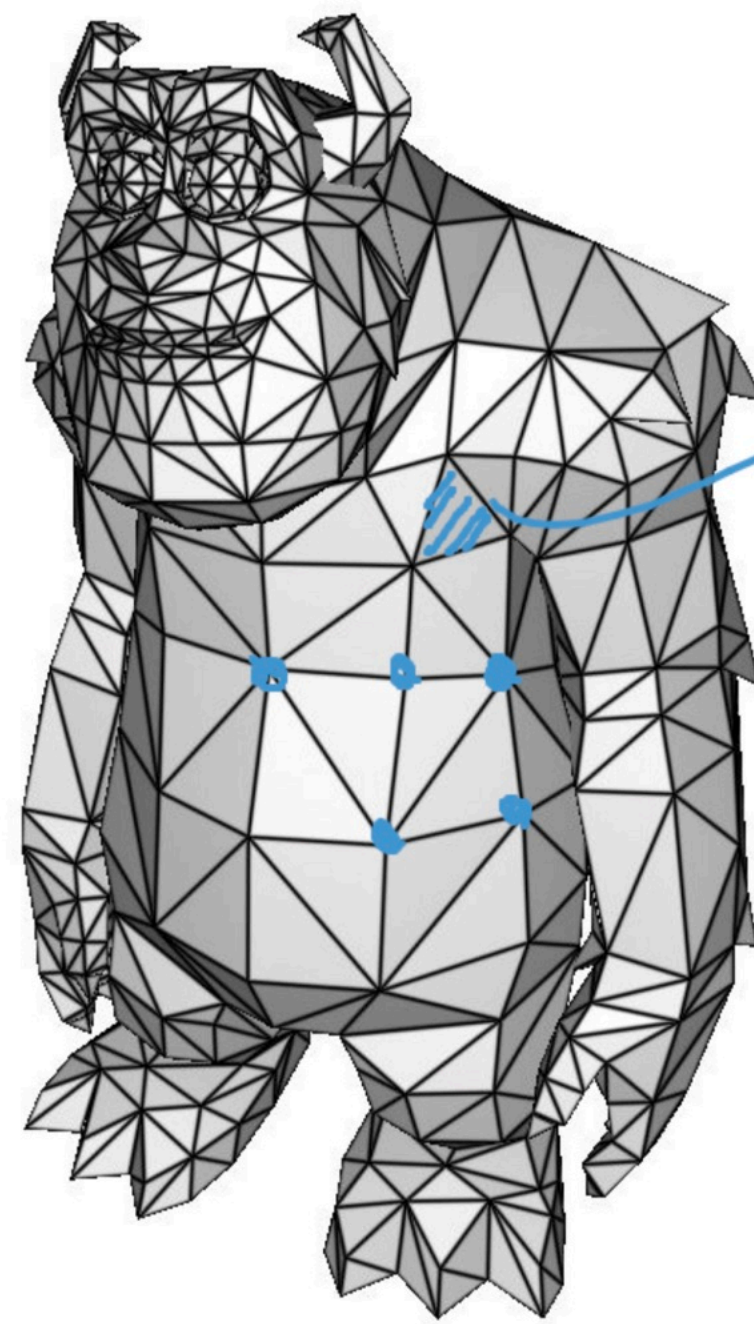


$\mathbb{R}^2$

- **set of integers:** { ....., −4, −3, −2, −1, 0, 1, 2, 3, 4, ... } $\mathbb{Z}$
- **set of natural numbers:** { 1, 2, 3, ... } $\mathbb{N}$
- **set of real numbers:** { 1.1, 3.14159265, $\sqrt{2}$, e, ... } $\mathbb{R}$

number of elements in a set A is called the cardinality, $|A|$

Set of triangles representing a character!



two sets: $\nearrow \mathbb{R}^3$

1. vertices
2. triangles

Describing sets: roster notation.

↳ explicitly listing all elements

* unordered
* unique

weekdays: $\{M, Tu, W, Th, F, Sa, Su\}$

positive even numbers up to (including) 10

E: $\{2, 4, 6, 8, 10\}$

element is in a set $\in$

$2 \in E$

element is not in a set $\notin$

$7 \notin E$

Python "in"

Describing sets: set-builder notation (uses predicates!).

↳ use a predicate to filter elements from some domain.

$$A = \{ \underbrace{x \in \mathbb{Z}}_{\text{domain}} \mid \underbrace{x > 0}_{\text{predicate}} \}$$

domain

predicate

"such that"

also use a colon :

$$B = \{ x^2 + 2 \mid x \in \mathbb{R} \}$$ also valid.

$$\mathbb{Q} = \{ \frac{x}{y} \mid x, y \in \mathbb{Z} \wedge y \neq 0 \}$$

set of rational numbers.

Practice: let $A = \{x^2 \mid x \text{ is even}\}$.

Which of the following are other ways to express A?
Discuss with peers and then vote on course webpage.

x % 2 == 0

A. $A = \{x^2 \mid x \bmod 2 = 0\}$

B. $A = \{x^2 \mid \frac{x}{2} \in \mathbb{Z}\}$

C. $A = \{(2x)^2 \mid x \in \mathbb{Z}\}$

D. None of the above.

E. $\boxed{\text{All of the above.}}$

n	$2n$	$2n+1$
1	2	3
2	4	5
3	6	7
4	8	9
5	10	11
⋮	⋮	⋮

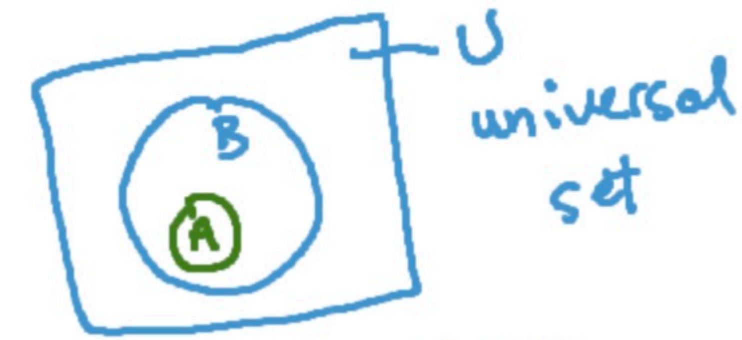
A B C D E ✋ ❤️ 👍 👎 😳 🤔 😊 🐦 🐢 🐍

1 1 1 1 115 0 0 0 0 1 2 0 10 0 4

Subsets and operations.

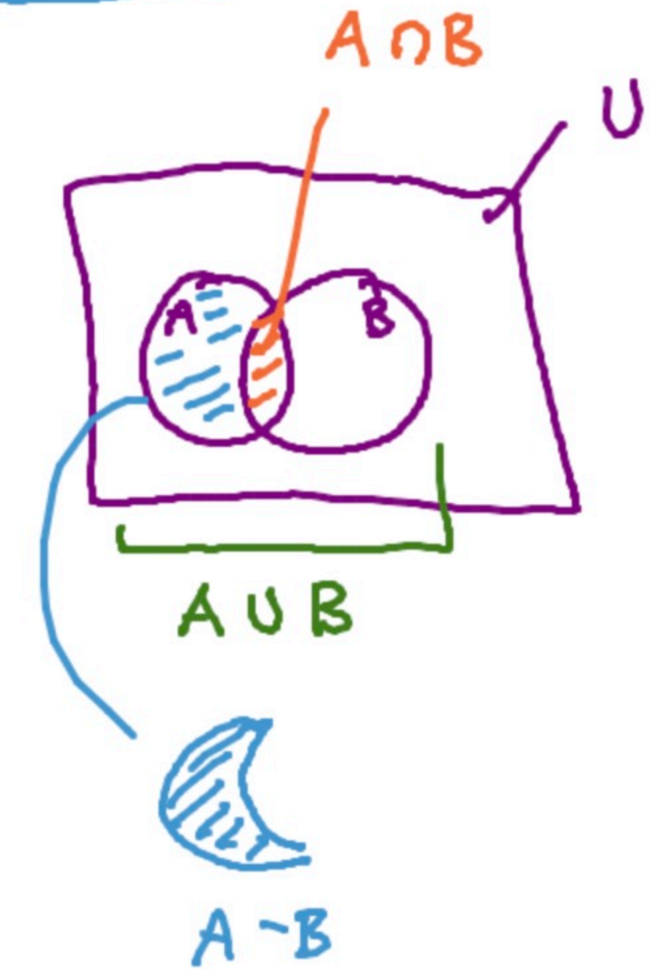
$A \subset B$ (strict) subset

$A \subseteq B$



Let A, B be sets.

e.g. let U = integers $\mathbb{Z}$
B = positive integers $\mathbb{Z}^+$
A = positive even integers



complement: $\overline{A} = U - A = \{x \in U \mid x \notin A\}$

union: $A \cup B = \{x \in U \mid x \in A \lor x \in B\}$

intersection: $A \cap B = \{x \in U \mid x \in A \land x \in B\}$

difference: $A - B = \{x \in U \mid x \in A \land x \notin B\}$

Example with counting sets.

In an in-class poll, there were three emoji options: 🐦, 🐢, 🐍. People may have clicked one or multiple emojis (everyone clicked at least one emoji). The poll results are below. Note that there is some overlap between those who clicked 🐦 with those who clicked 🐦🐢 as well as those who clicked all three (🐦 🐢 🐍). How many people were in class?

B 🐦	T 🐢	S 🐍	BT 🐦🐢	BS 🐦🐍	TS 🐢🐍	BTS 🐦🐢🐍
~~4~~ 11	~~3~~ 12	~~5~~ 9	~~2~~ 8	~~[illegible]~~ 7	~~2~~ 8	6

$$|B \cup T \cup S| = |B| + |T| + |S| - |B \cap T| - |B \cap S| - |T \cap S| + |B \cap S \cap T|$$