Middlebury

CSCI 200: Math Foundations of Computing

Spring 2026

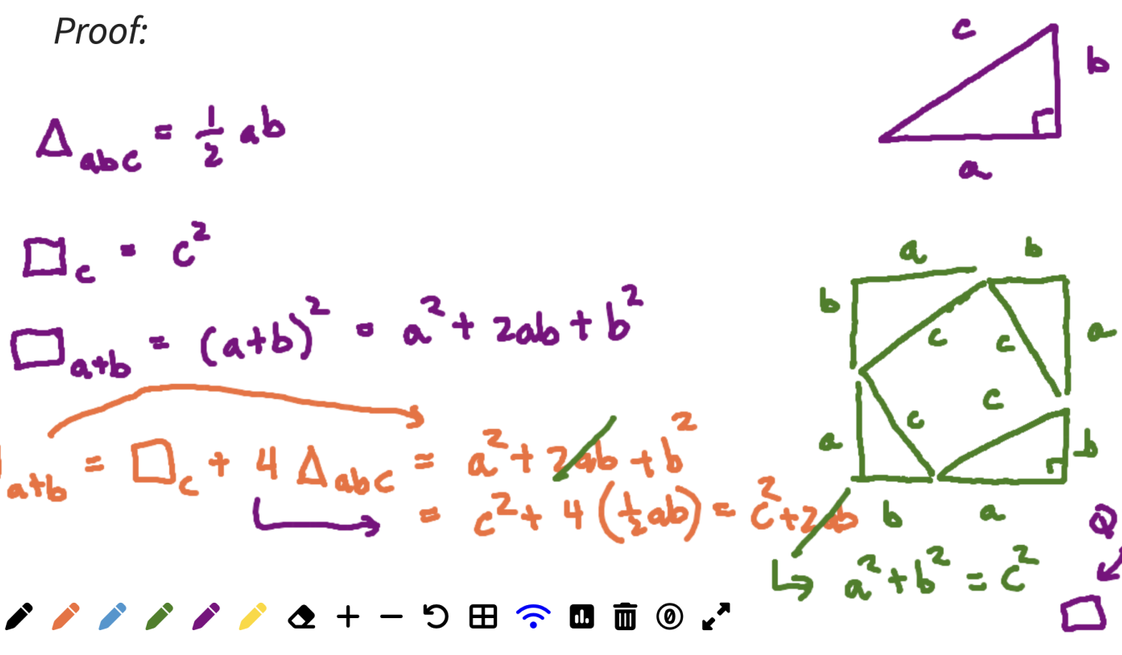
Lecture 1W: Logic

Last time, we introduced proofs.

A **mathematical proof** is a verification of a **proposition** by a chain of logical **deductions** from a set of **axioms**.

Theorem: Given a planar right triangle with side lengths a, b and c (meaning there is one angle of 90°, which we will take to be opposite the side with length c), then $c^2 = a^2 + b^2$.

Proof:



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Goals for today:

By the end of today's lecture, you will be able to:

- Create **compound statements** with $\wedge$, $\vee$ and $\neg$, and implications ($\implies$, $\iff$).
- Prove compound statements using a **truth table**.

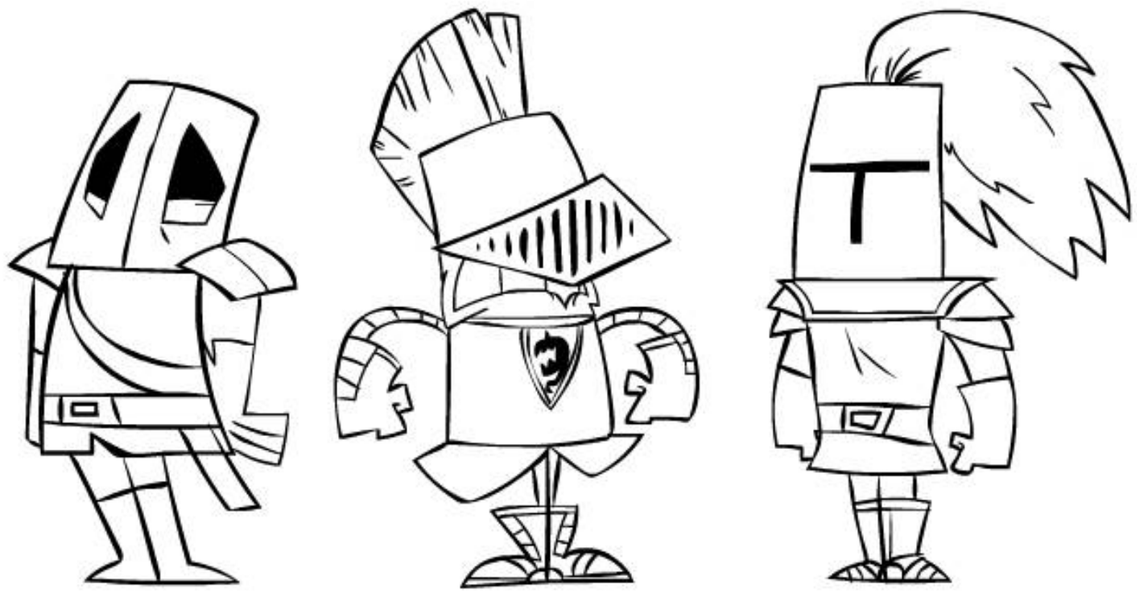


Image credit

But first! A logic puzzle...

You encounter three trolls which are either knights (always tell the truth) or knaves (always lie). They let you pass if you can identify who is a knight and who is a knave. Let's just start with Troll A. Can Troll A be a knight?

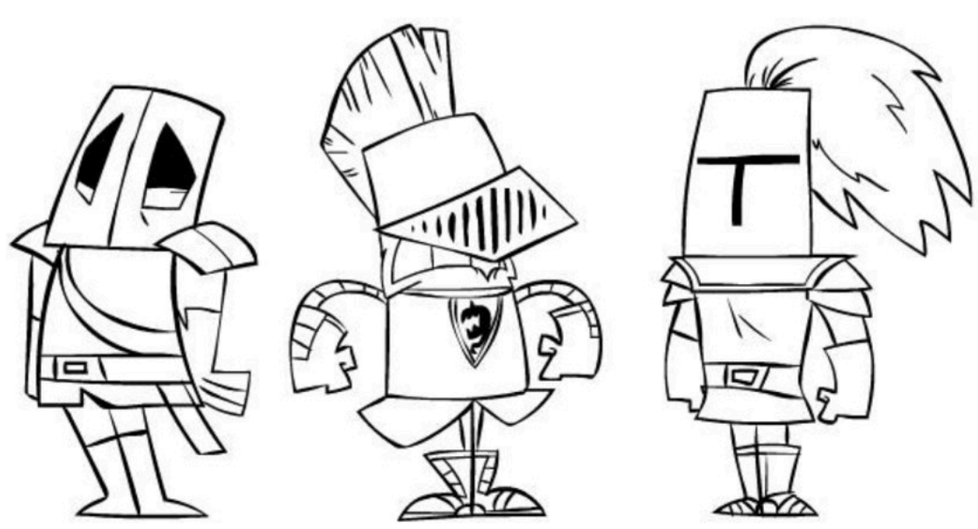
F ?

Suppose A is a knight

- **Troll A:** Either we are all knaves or two of us are knights.
- **Troll B:** If I am a knight, then the other two are knaves.
- **Troll C:** Troll B is lying.

Troll B be a knight? X

Troll B be a knave? X



Troll A is a knave

Logical operators: $\wedge$, $\vee$, $\neg$

Let p = *I am wearing a blue shirt today.*
Let q = *Today is Monday.*

$p \wedge q$: I am wearing a blue shirt AND today is Monday.

$p \vee q$: I am wearing a blue shirt OR today is Monday

$\neg p$: I am NOT wearing a blue shirt

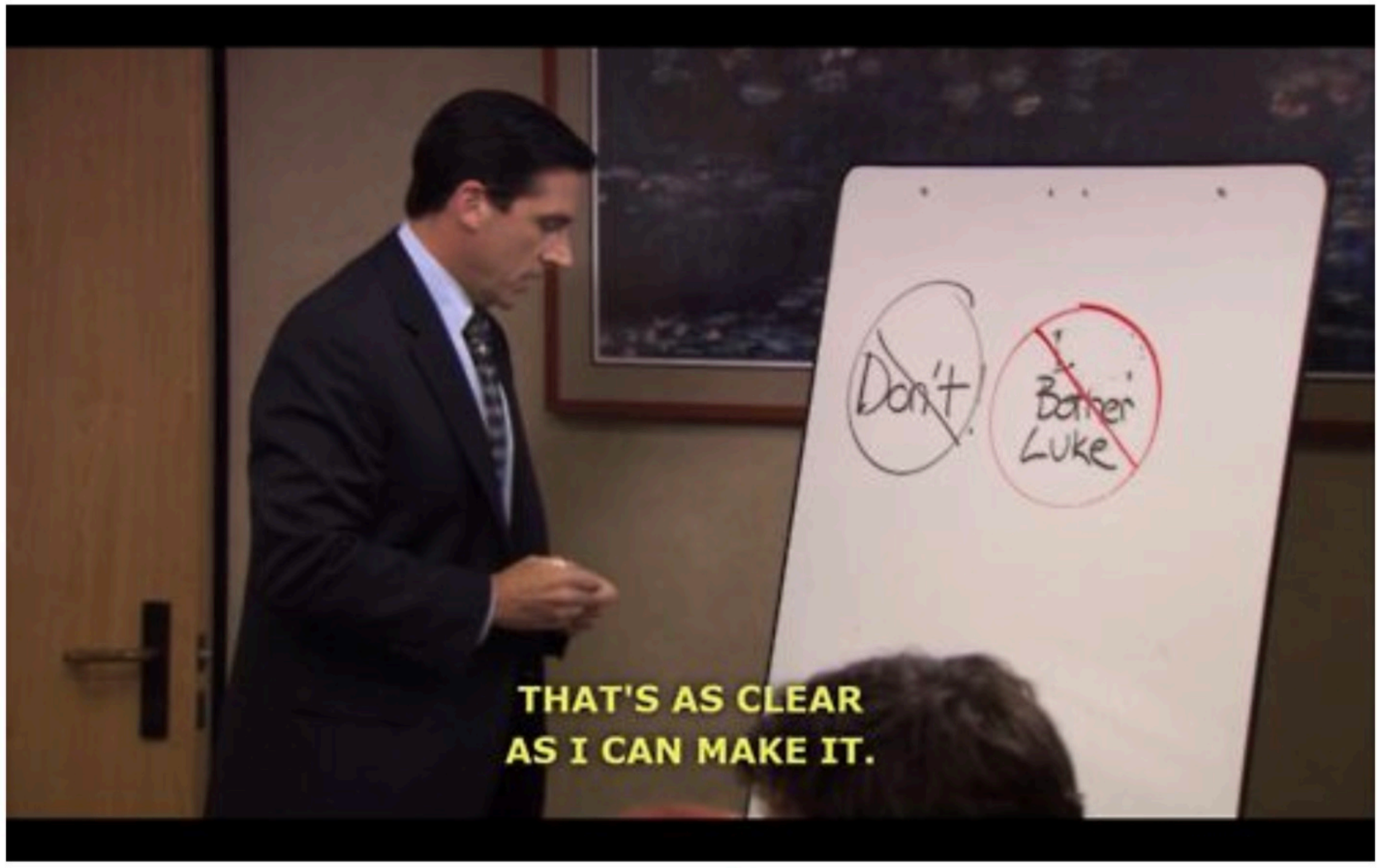
p	q	$p \wedge q$	$p \vee q$	$\neg p$
T	T	T	T	F
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

This notation is useful to avoid situations like this...

$\wedge$ AND

$\vee$ OR

$\neg$ NOT



Implications: think of a "promise".

Let p = *You get 100% on the final.*

Let q = *You get an A in the course.*

"if p, then q"

$p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

→ is the promise valid

Is this statement True or False? Vote at *Reactions* page on course website.

If $1 + 1 = 3$ then I am a dinosaur.

A. True
B. False

A B C D E ✋ ❤️ 👍 👎 😳 🤔 😊 🦆 🐢 🐍

26 8 0 0 0 0 0 2 0 0 0 0 0 9 0

Biconditional: "if-and-only-if"

$$p \iff q = (p \to q) \wedge (q \to p)$$

p	q	p → q	q → p	p ↔ q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Proving with a truth table.

$(p \to q) \vee (q \to r)$]

- Identify and label smaller propositions (with variables).
- Write overall proposition symbolically.
- Make a column for each variable and build up to overall proposition.

2^3 rows

Example: Prove the following: *If you eat spinach everyday, then you will win the lottery, or if you win the lottery, you will lose your job.*

p: *you eat spinach everyday*; q: *you will win the lottery*; r: *you will lose your job*

p	q	r	$(p \to q) \vee (q \to r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

all rows are true ✓

Back to our original puzzle.

T → T is T
F → F is T
?

$p \leftrightarrow t_1$
$q \leftrightarrow t_2$
$r \leftrightarrow t_3$
all must be true

Let p, q, r represent whether Trolls A, B, C are a knight (True) or a knave (False). We already know $\neg p$ is true (Troll A is a knave).

- **Troll A:** Either we are all knaves or two of us are knights.

$$t_1 = (\neg p \wedge \neg q \wedge \neg r) \vee ((p \wedge q) \vee (p \wedge r) \vee (q \wedge r)) \equiv (\neg q \wedge \neg r) \vee (q \wedge r)$$

- **Troll B:** If I am a knight, then the other two are knaves.

$$t_2 = q \implies (\underbrace{\neg p}_{T} \wedge \neg r) \equiv q \rightarrow \neg r$$

- **Troll C:** Troll B is lying.

$$t_3 = \neg q \qquad t_3 = \neg t_2$$

- **Consistency:**

$$c = (p \iff t_1) \wedge (q \iff t_2) \wedge (r \iff t_3)$$

q	r	t_1	t_2	t_3	c
T	T	F	T	F	T ✓
T	F				
F	T				
F	F				

?

A: knave
B: knight
C: knave

de Morgan's laws: a tool to simplify compound propositions.

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Summary & Reminders

- Problem Set 1 due (on paper) on Friday at 9:45am: Quiz 1 will ask you to re-solve one of these problems.
  - Because it's the first quiz, I'm telling you which problem I will ask: I am going to ask you to solve Problem 0.8. For (b), why is $x = 7$ not a solution?
  - But you must show all your work: 4/5 for each part if only the answer is provided, but no work is shown.
  - Come visit during office hours if you have questions!
    General office hours still TBD, but for now:
    - **Tomorrow (Thursday):** 1pm - 3pm