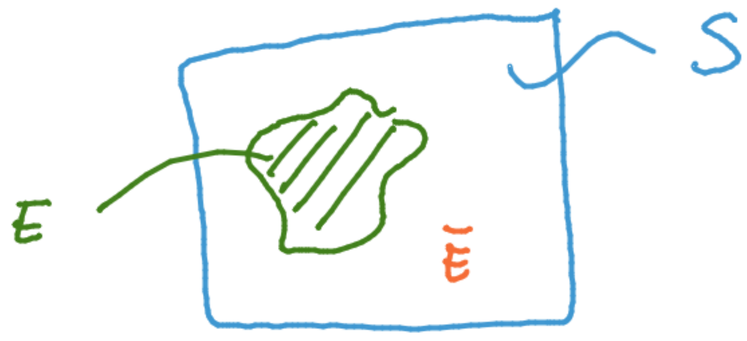
Recap of last time (probability definitions).

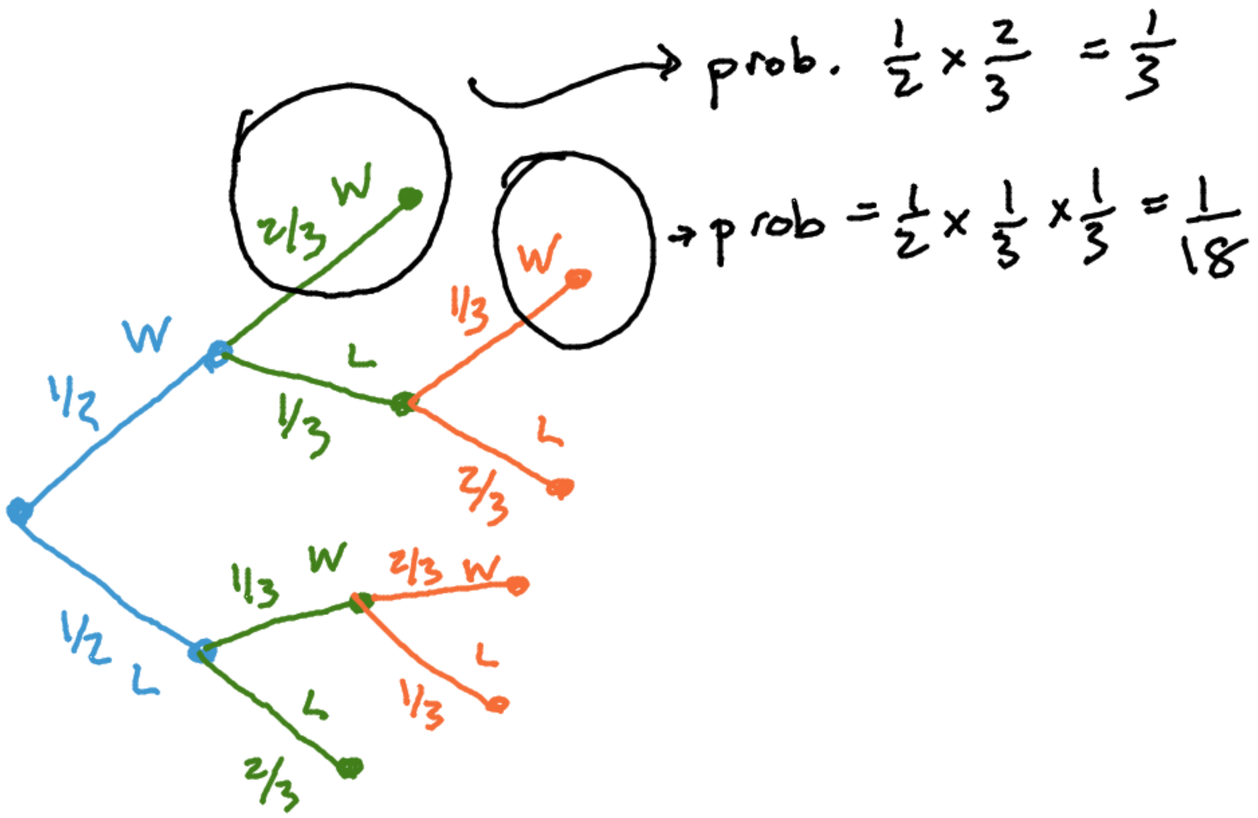
- State your assumptions!
- Identify sample space (S) and event space (E).
- Draw probability tree to compute outcome probabilities (if possible).
- Outcomes have equal probabilities? $p(E) = \frac{|E|}{|S|}$
- Outcomes have unequal probabilities? $p(E) = \sum_{e \in E} p(e)$
- Probability of E not happening? $p(\overline{E}) = 1 - p(E)$



Exercise 0: draw the complete probability tree for a best 2-out-of-3 series.

Assumptions:

- Each person has a 1/2 chance of winning the first game.
- Chance of winning after a win is 2/3 (so chance of winning after a loss is 1/3).



probability of winning series given that we win 1st game

$= \frac{1}{3} + \frac{1}{18} = \frac{7}{18}$???

does this make sense?

hmm... missing something...

what is the chance of winning the series *given* that you win the first game?

Conditional probability: goal is to answer "what is the probability of A *given* B?"

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\text{probability of } \underline{\text{both}} \text{ A and B}}{\text{probability of B}}$$

"given"

independence: A, B independent events iff $P(A \cap B) = P(A) \cdot P(B)$ (1)

to show? calculate $P(A \cap B)$, $P(A)$, $P(B)$ + check (1)

e.g: roll die every day of week

outcomes to get ⚄ on Tuesday = 1

total # outcomes = 42

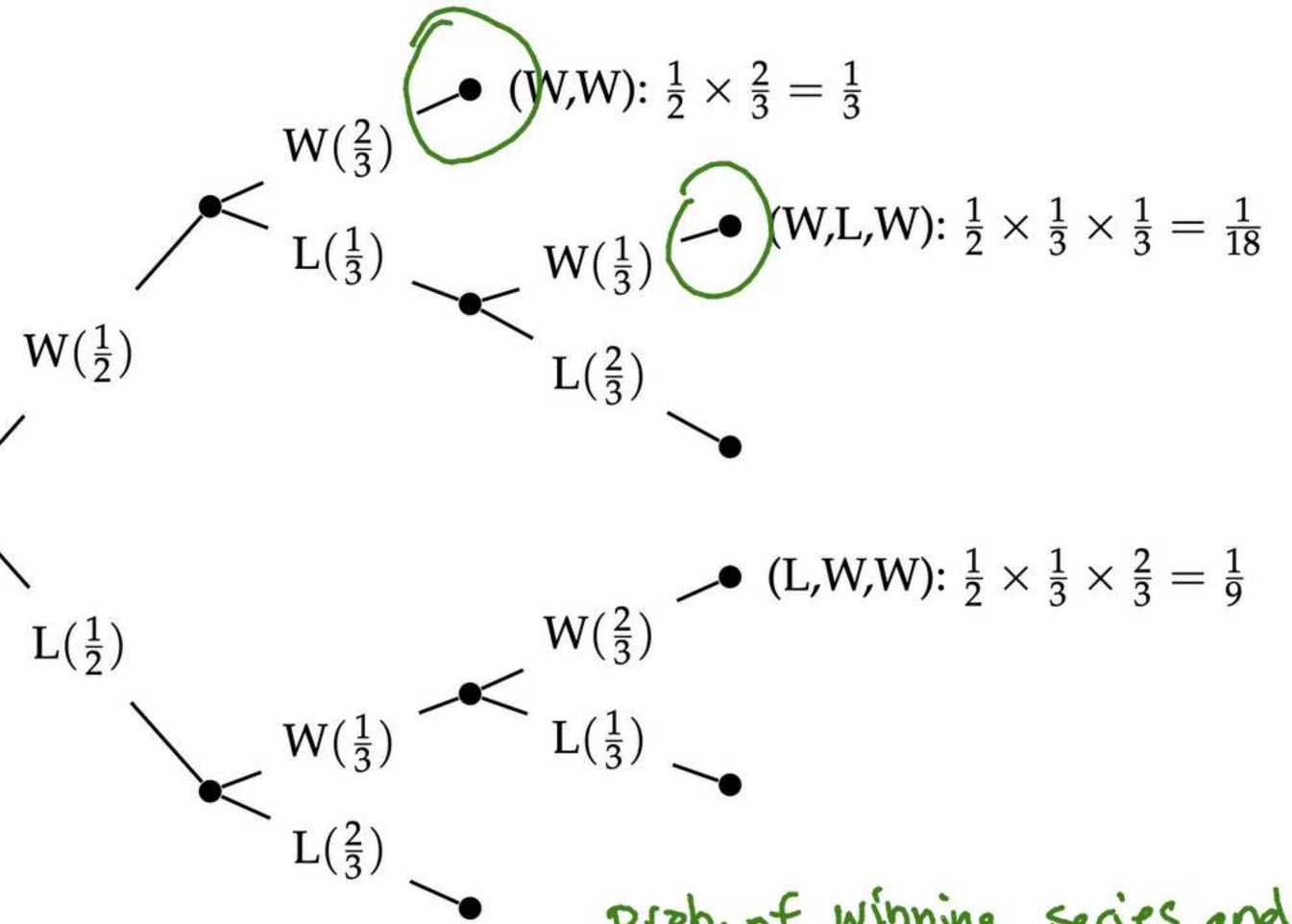
prob. of getting ⚄ on Tuesday = $\frac{1}{42}$

$P(A) = \frac{1}{7}$

$P(B) = \frac{1}{6}$

$\frac{1}{7} \cdot \frac{1}{6} = \frac{1}{42}$

Probability of winning the series *given* that you win the first game.



$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{7/18}{1/2} = 7/9$ ←

prob. of winning series and we won 1st game.
= 7/18

prob. of winning 1st game = $\frac{1}{2}$

Exercise 1: rolling two dice.

$E = \{(5,5),(4,6),(6,4)\}$

$P = \frac{|E|}{|S|} = \frac{3}{36} = \frac{1}{12}$

1. What is the probability that the sum of both die values adds to 10?

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- 1/25
- 10/36
- 1/12
- 1/10

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Sample space: all possible outcomes

$(1,1),(1,2),(1,3)\ldots(1,6)$

$(2,1),(2,2),\ldots(2,6)$

$(6,1),(6,2),\ldots(6,6)$

6 × 6 = 36 outcomes

2. What is the probability that the sum of both dice is 10, given at least one die is a 5?

different ways to do this!

1) how many ways to get [⚄]? 1 way, 11 ways

$\{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(1,5),(2,5),(3,5),(4,5),(6,5)\}$

$P = \frac{1}{11}$

2) using conditional probability

#possible outcomes = 36

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/36}{11/36} = \boxed{\frac{1}{11}}$

A: sum = 10; B: at least 1 [⚄]

Exercise 1: rolling two dice.

1. What is the probability that the sum of both die values adds to 10?

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- 1/25
- 10/36
- 1/12
- 1/10

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2. What is the probability that the sum of both dice is 10, given at least one die is a 5?

ways to not get a [⚄]? 25 outcomes → prob. = $\frac{25}{36}$

prob. of getting a 5 = $1 - \frac{25}{36} = \frac{11}{36}$

Exercise 2: rolling bit strings of length 4.

A: event in which you randomly create bit strings of length 4 that begin with a 1.

B: event in which you randomly create bit strings of length 4 with an even number of 1's.

Are A and B independent?

$|S| = 2^4 = 16$

$p(A \cap B) \stackrel{?}{=} p(A) \cdot p(B)$

A:

1000

3

2

$= 8$

B:

1100 ←
0000
1111 ←
0011
1001 ←
1010 ←
0101
0110

8

$p(A) = \frac{8}{16} = \frac{1}{2}$

$p(B) = \frac{8}{16} = \frac{1}{2}$

$p(A \cap B) = \frac{4}{16} = \frac{1}{4}$

$\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$ ✓ independent.