Recap of the Tree method for divide & conquer recurrence relations.

subproblems

how much work is done outside of recursive calls

general recurrence: $f(n) = a \cdot f(\frac{n}{b}) + \underbrace{c \cdot n^d}$

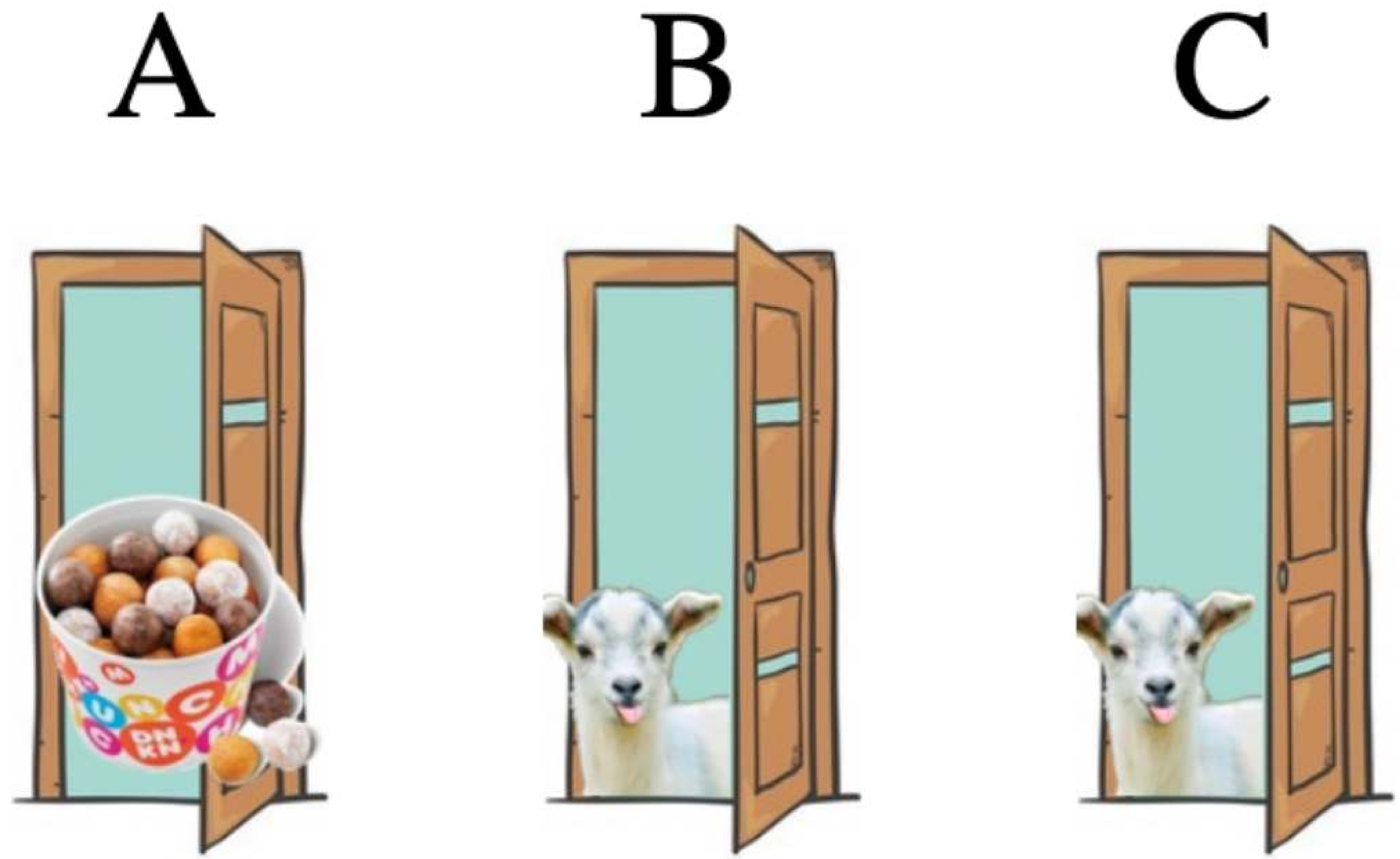
factor the problem size shrinks during recursive calls

merge-sort: $f(n) = 2 f(\frac{n}{2}) + n - 1$

binary search: $f(n) = f(\frac{n}{2}) + c$

$$f(n) = \begin{cases} O(n^d) & \text{if } a < b^d \\ O(n^d \log n) & \text{if } a = b^d \\ O(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

Pick the right door and I will bring munchkins to class on Friday for this section!

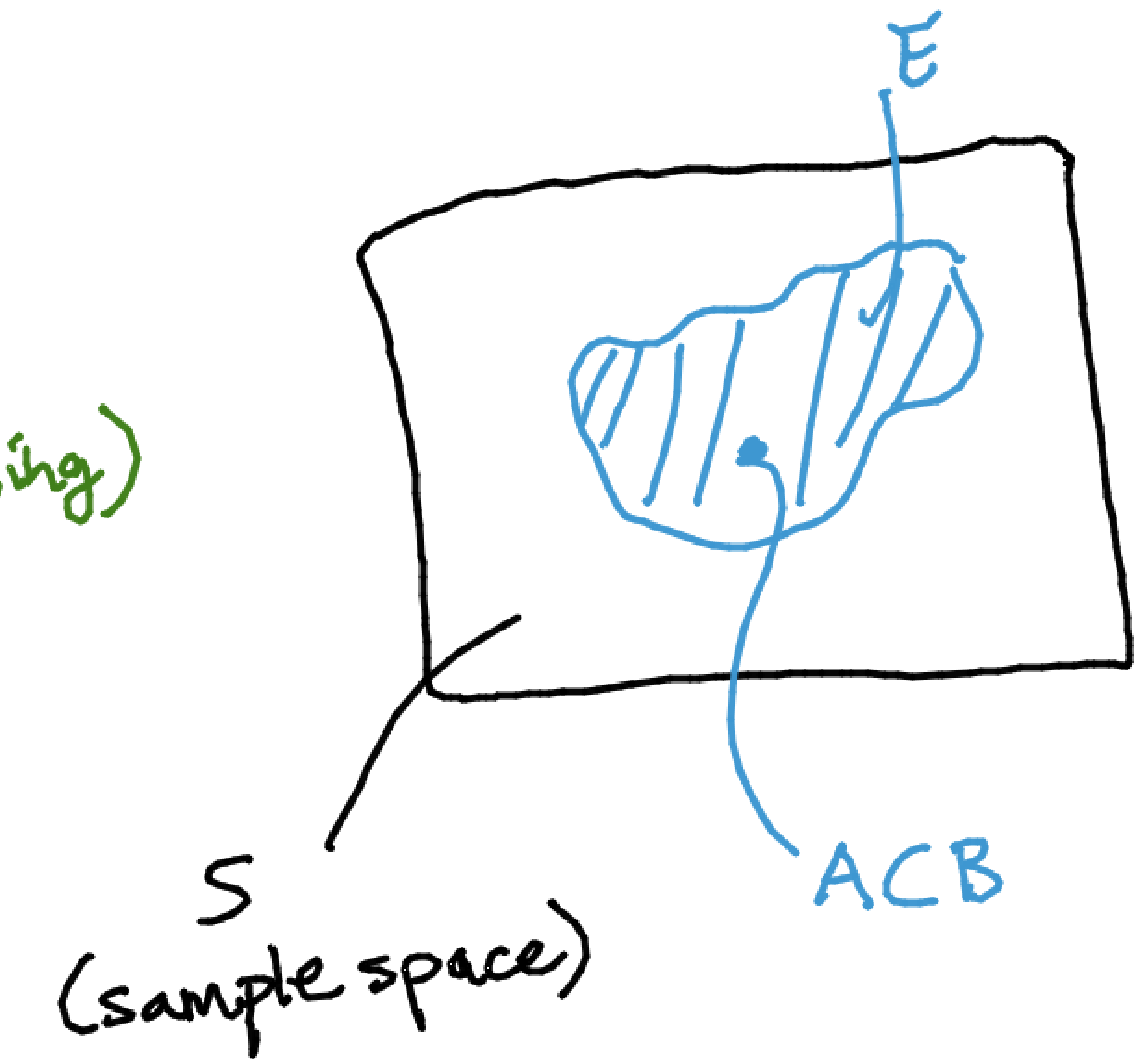


A few definitions before we can analyze this.

- **outcome (a.k.a. sample point):** information about some experiment
  - host picks door
  - player picks door
  - host reveals a door with a goat

assume we always switch doors.

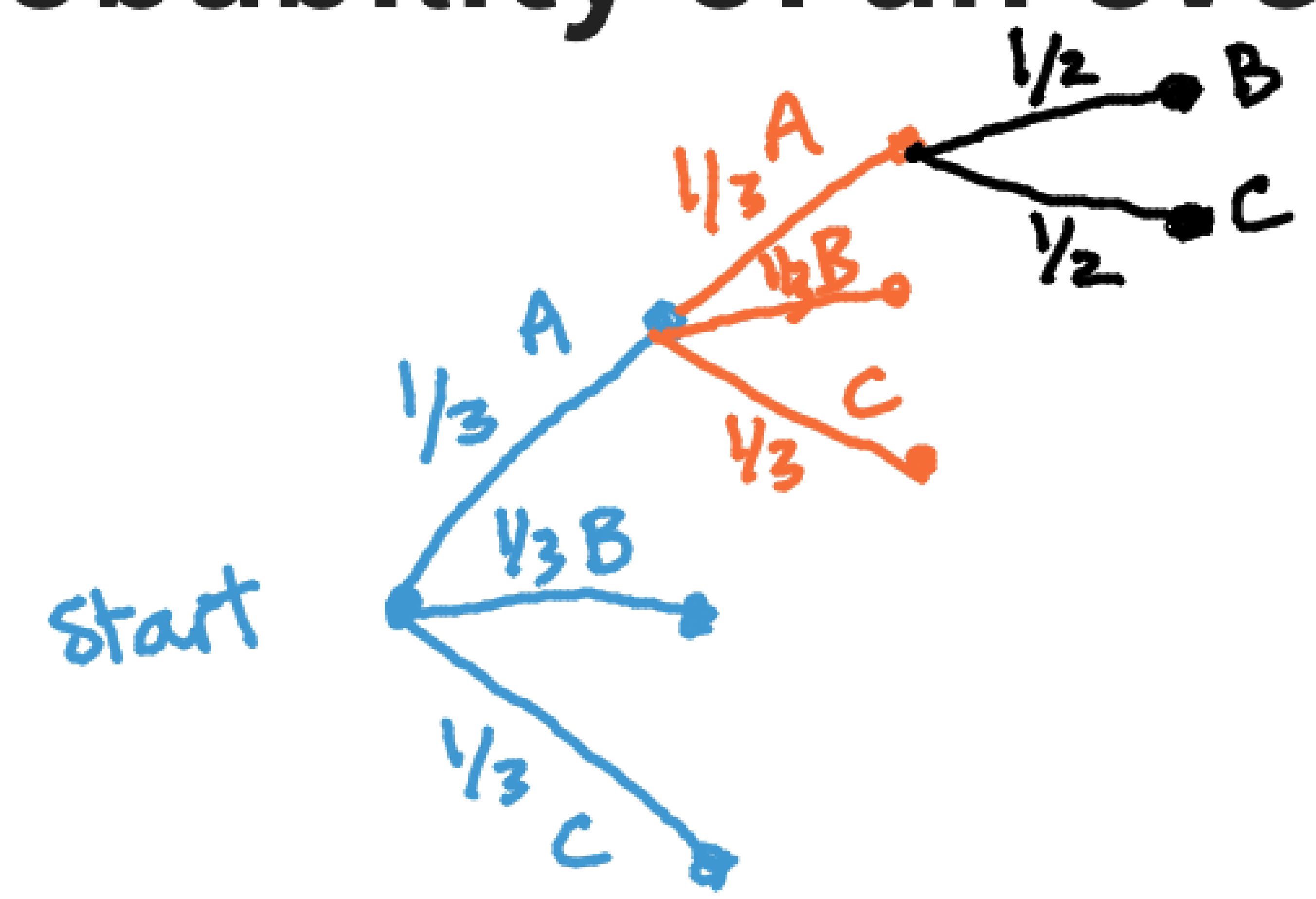
- **sample space:** set of all possible outcomes.
- **event:** subset of sample space.
  (something we're interested in, e.g. winning)



Steps to calculate the probability of an event.

S $|S|$

1. Find the **sample space**.



E

2. Identity the **event(s)** of interest.

3. Determine the **outcome probabilities** (*multiply* edge weights).

4. Compute **event probability** by *adding* probabilities of outcomes in the event.

if all outcomes have <u>equal</u> probability

$$p(E) = \frac{|E|}{|S|}$$

if outcomes have <u>unequal</u> probability

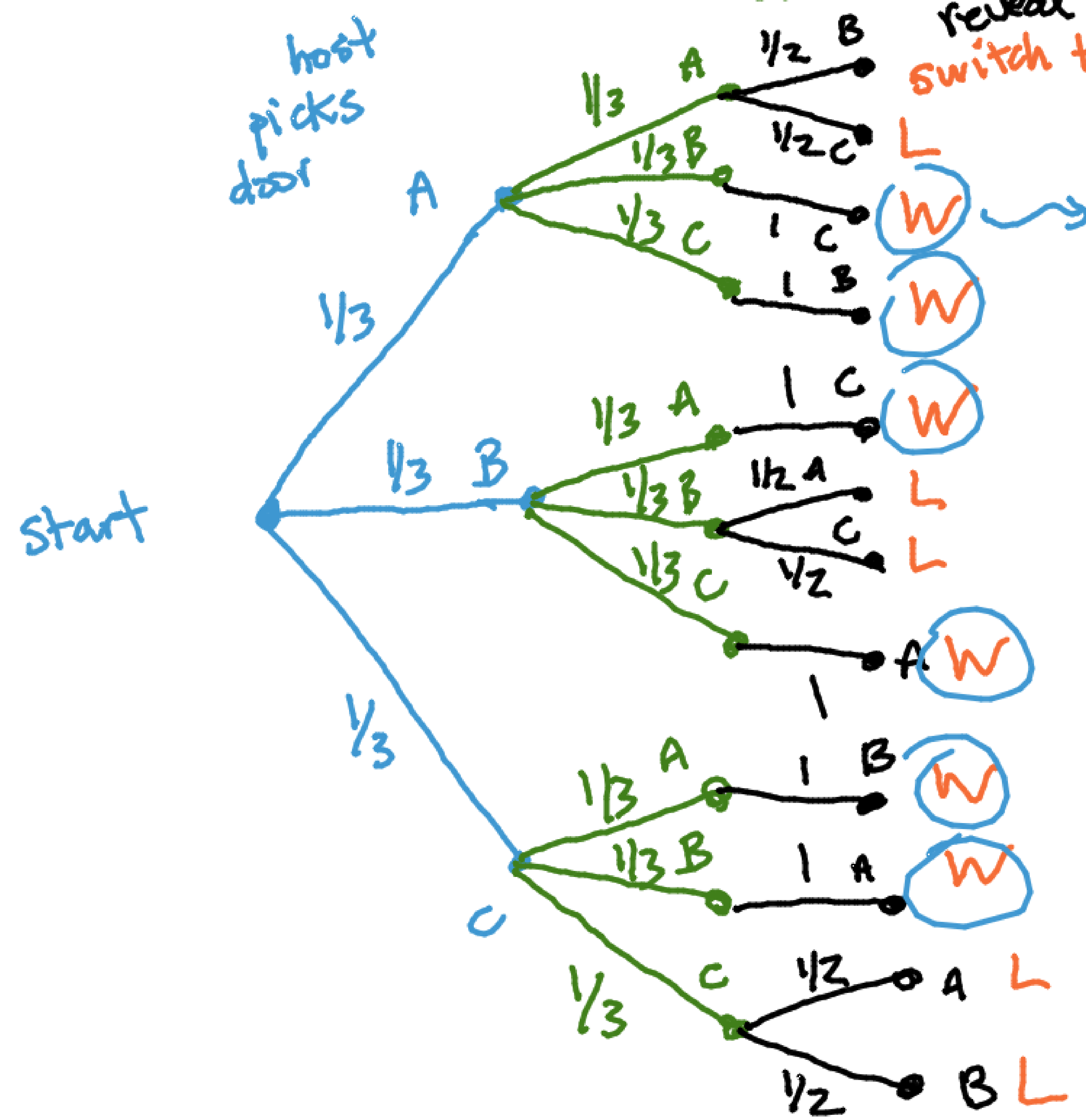
$$p(E) = \sum_{e \in E} p(e)$$

If you always switch doors, what is the probability that you win?

state your assumptions

- Host picks a door with probability 1/3.
- Player picks a door with probability 1/3.
- Host reveals other door with probability 1/2 (if there is a choice).

host reveal a door with goat.



probability $= \frac{1}{3} \times \frac{1}{3} \times 1 = \frac{1}{9}$

all W probabilities are $\frac{1}{9}$

$$P(E) = \frac{1}{9} \times 6 = \frac{2}{3}$$

$(\approx 66.67\%)$

always switch doors!

Exercise 1: probability of getting a 5-card straight flush?

Straight flush: same suit, values are *consecutive*.
examples: (A, 2, 3, 4, 5), (4, 5, 6, 7, 8), (10, J, Q, K, A), but **not** (Q, K, A, 2, 3).

$|S|$? $|E|$? $P(E) = \frac{|E|}{|S|}$

$\underbrace{A\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10}_{10 \text{ options}}\ J\ Q\ K$

possible 5-card hands $\binom{52}{5} = |S|$

values to start flush 10

suits 4

$|E| = 4 \times 10 = 40$

$P(E) = \frac{40}{\binom{52}{5}} = 0.00154\,\%$

Exercise 2: probability of correctly picking 6 distinct lottery numbers (out of 55 possible numbers)?

- (a) order of numbers does **not** matter?

- (b) order of numbers *does* matter?

(permutation)

$P(n,k) : \frac{n!}{(n-k)!}$

$C(n,k) = \frac{n!}{(n-k)!k!}$

(combination)

Select the correct response for each question (a) and (b) (slido.com #2628175)

24

- (a) 1 / P(55, 6); (b) 1 / C(55, 6)
- (a) 1 / P(55, 6); (b) 1 / P(55, 6)
- (a) 1 / C(55, 6); (b) 1 / C(55, 6)
- (a) 1 / C(55, 6); (b) 1 / P(55, 6)
- None of these.

Voting as Anonymous

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