2928. Distribute Candies Among Children I

Solved

Easy | Topics | Companies | Hint

You are given two positive integers `n` and `limit`.

Return *the **total number** of ways to distribute* `n` *candies among* `3` *children such that no child gets more than* `limit` *candies.*

Example 1:

```
Input: n = 5, limit = 2
Output: 3
Explanation: There are 3 ways to distribute 5 candies such that no child gets more than 2 candies: (1, 2, 2), (2, 1, 2) and (2, 2, 1).
```

Example 2:

```
Input: n = 3, limit = 3
Output: 10
Explanation: There are 10 ways to distribute 3 candies such that no child gets more than 3 candies: (0, 0, 3), (0, 1, 2), (0, 2, 1), (0, 3, 0), (1, 0, 2), (1, 1, 1), (1, 2, 0), (2, 0, 1), (2, 1, 0) and (3, 0, 0).
```

Constraints:

- `1 <= n <= 50`
- `1 <= limit <= 50`

Possible solution to the candy distribution problem.

```python
def distribute_candies(n: int, limit: int) -> int:
  """Counts the number of ways to distribute 'n' candies
     to 3 people so that each person has a maximum
     of 'limit' candies."""
  n_ways = 0
  for i in range(limit + 1):
      for j in range(limit + 1):
          for k in range(limit + 1):
              if i + j + k == n:
                  n_ways += 1
                  # print(i, j, k)
  return n_ways
```

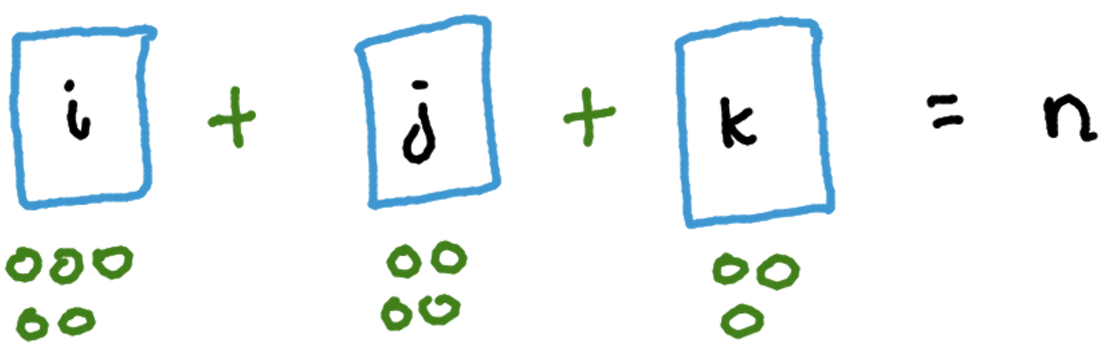
Try $n = \ell = 50$: how much time does this take?
What about $n = \ell = 100$? $n = \ell = 250$? $n = \ell = 500$?

Our problem for today: in how many ways can you distribute n candies to 3 people if every person is limited to ℓ candies.

Breaking the problem down:

1. In how many ways can you distribute n candies to $k = 3$ people?
2. Then subtract ways that go over the limit ℓ.

how many +'s for k people? k-1



stars ↑ ↗ dividers (bars)

ooooo+oooo+oo ← 2 options for each slot either a o or a +

in how many ways can we pick location of (k-1) +'s (bars) from a total (n+k-1) slots $\binom{n+k-1}{k-1}$ "stars and bars"

Our problem for today: in how many ways can you distribute n candies to 3 people if every person is limited to ℓ candies.

$\binom{n+k-1}{k-1}$

"stars and bars function"

Breaking the problem down:

1. In how many ways can you distribute n candies to $k = 3$ people?
2. Then subtract ways that go over the limit ℓ.

we have total # ways = sb(n, k)

1.) suppose we give $(\ell+1)$ candies to 1 person: $n-(\ell+1)$ left

how many ways to pick 1 person? $\binom{3}{1} = 3$

$- 3 \times sb(n-(\ell+1), 3)$

2) add back in # ways in which 2 people over limit; $n-2(\ell+1)$ left

$\binom{3}{2} = 3$ $\quad + 3 \times sb(n-2(\ell+1), 3)$

3) subtract # ways in which 3 people over limit: $n-3(\ell+1)$ left

$\binom{3}{3} = 1$ $\quad - sb(n-3(\ell+1), 3)$

```
sb(n, k):
    check if n < 0
        return 0
    math.comb(n+k-1, k-1)
```

Possible solution to the candy distribution problem.

```python
def stars_and_bars(n: int, k: int) -> int:
    """Returns the number of ways 'n' objects can
       be distributed to 'k' slots."""
    if n < 0:
      return 0
    return math.comb(n + k - 1, k - 1)

def distribute_candies(n: int, limit: int) -> int:
    """Counts the number of ways to distribute 'n' candies
    to 3 people so that each person has a max of limit"""
    n_ways = stars_and_bars(n, 3)
    n_ways -= 3 * stars_and_bars(n -     (limit + 1), 3)
    n_ways += 3 * stars_and_bars(n - 2 * (limit + 1), 3)
    n_ways -=     stars_and_bars(n - 3 * (limit + 1), 3)
    return n_ways
```

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