Announcements:

- Office hours change for today (04/03) & next Wednesday (04/10): **3:30 - 4:30pm**

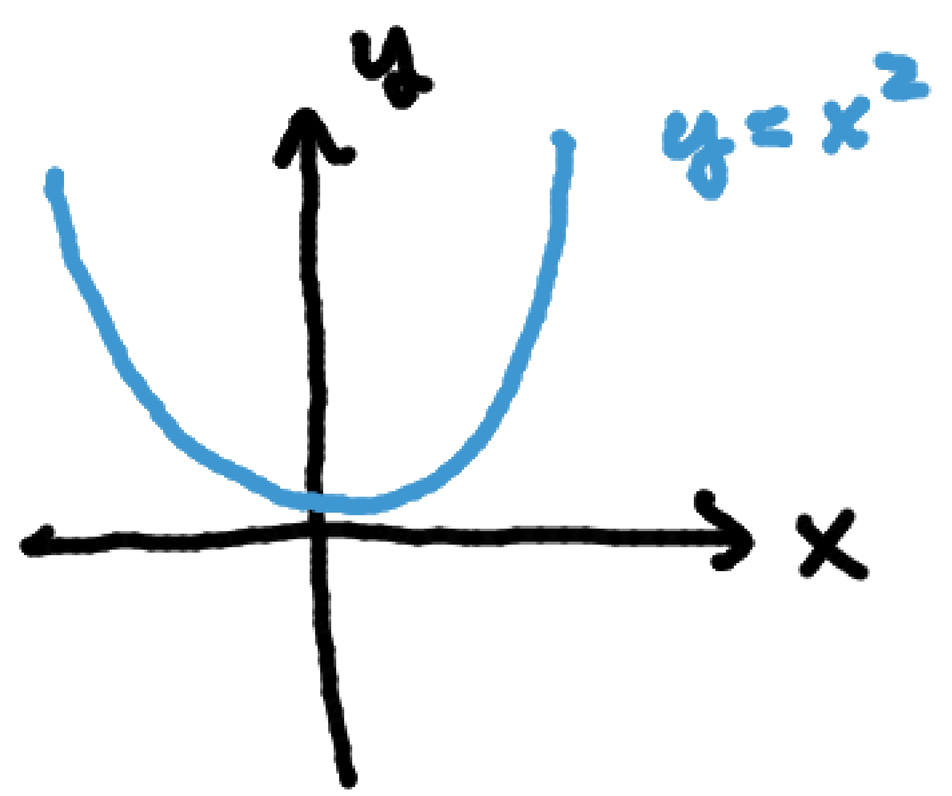
- Plan for Monday 04/08 (**ECLIPSE DAY**): optional class - I will post a **video** this weekend and will create a **worksheet**. If you want to come to class for **practice**, please watch the video before class. Solutions to the worksheet will be posted after class. Office hours on 04/08: 11am - 12pm.

  I will send a reminder email with the video link this weekend.

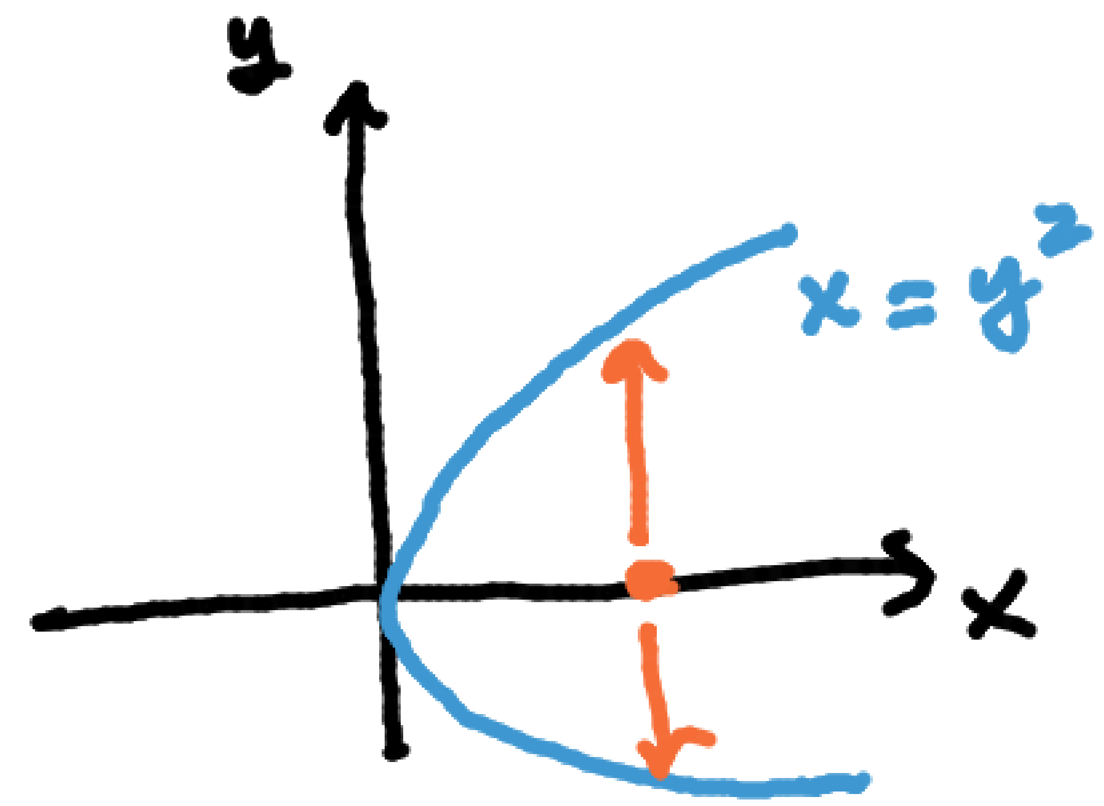
- Please remember to do your problem set self-assessments! Also, ensure your equations render correctly.

Phases of a Total Solar Eclipse (Artistic Rendering) (Credit: Catherine Miller)

A function is a map from sets $A \to B$ such that *every* element of A maps to a *unique* element of B.

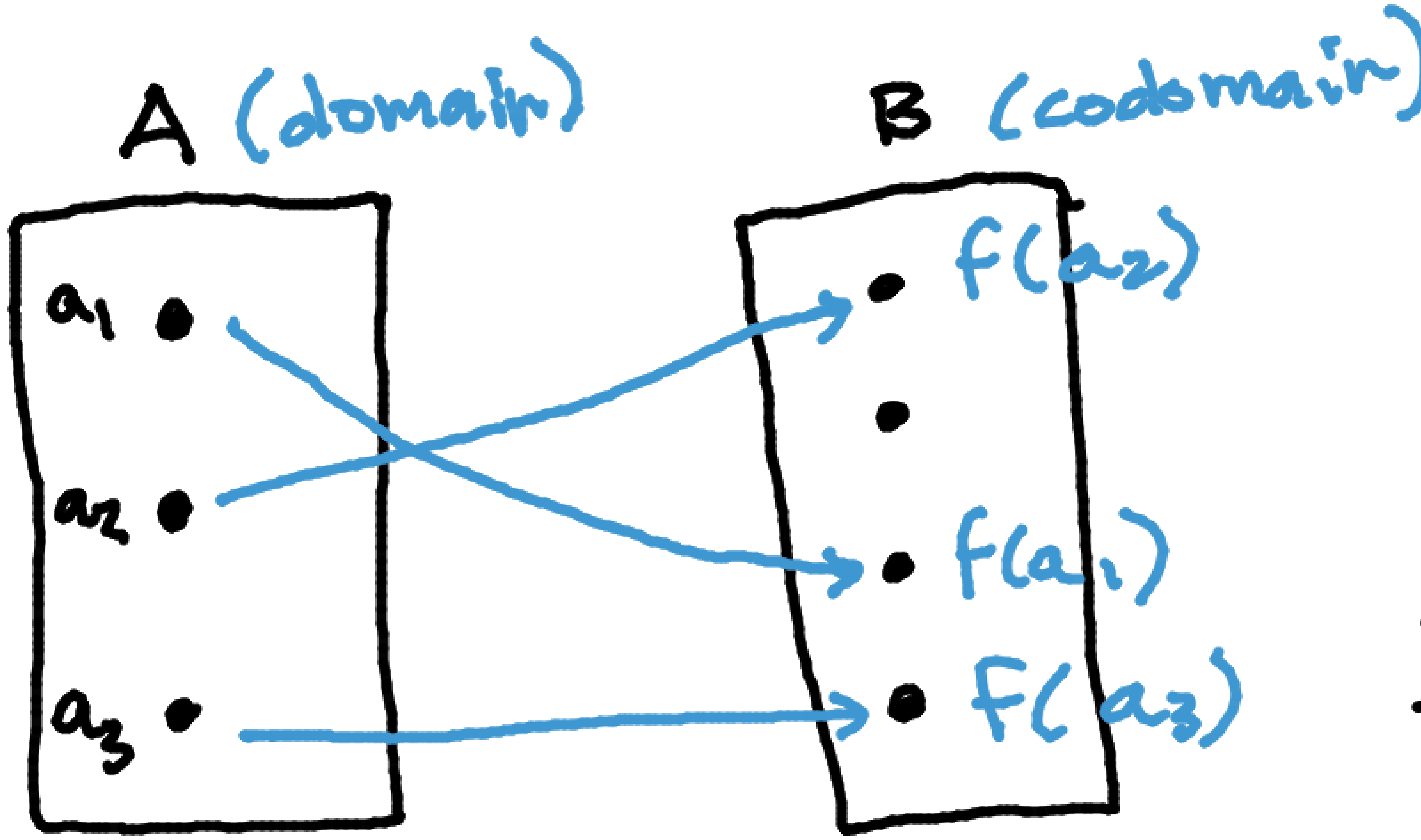


function from x to y



not a function from x to y

$$\underbrace{f}_{\text{function}} : \overbrace{A}^{\text{domain}} \to \overbrace{B}^{\text{codomain}}$$



$$f(a) = \overbrace{b}^{\text{image of } a}$$

$$f^{-1}(b) = a$$

a is preimage of b

range: set of all images

$$\{f(a_1), f(a_2), f(a_3)\}$$

Exercise 1: identify the domain, codomain and range of the following functions.

1. $f\colon \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 2n$.

domain codomain

range: set of even numbers.

2. $g\colon \{1, 2, 3\} \to \{a, b, c\}$ defined by $g(1) = c$, $g(2) = a$ and $g(3) = a$.

domain codomain

range: $\{a, c\}$

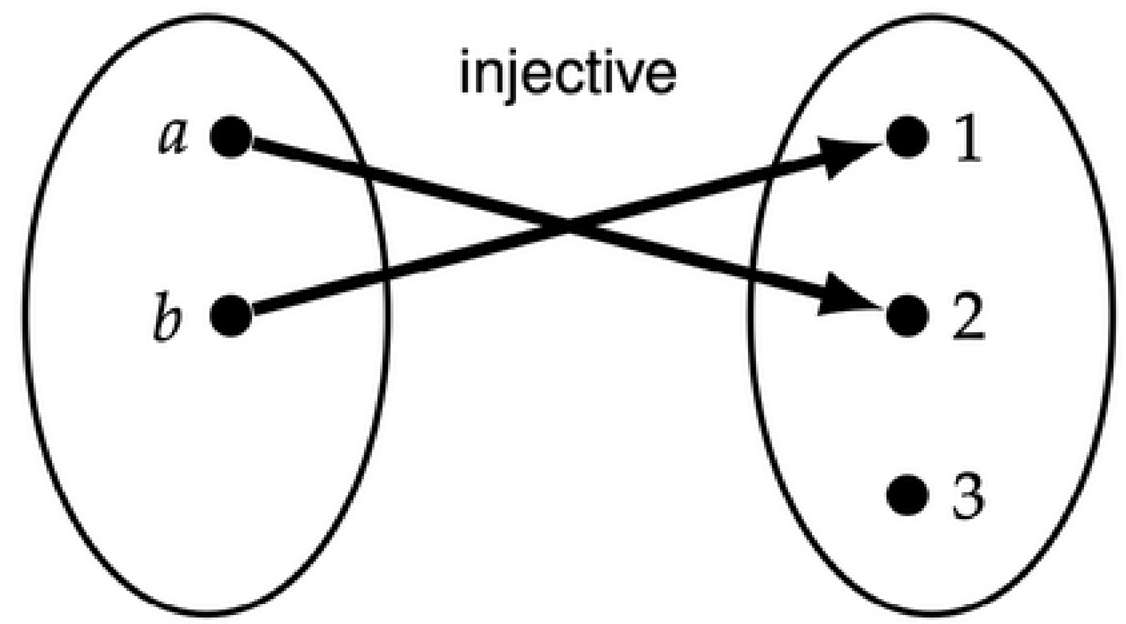
Properties of functions: injective, surjective, bijective.

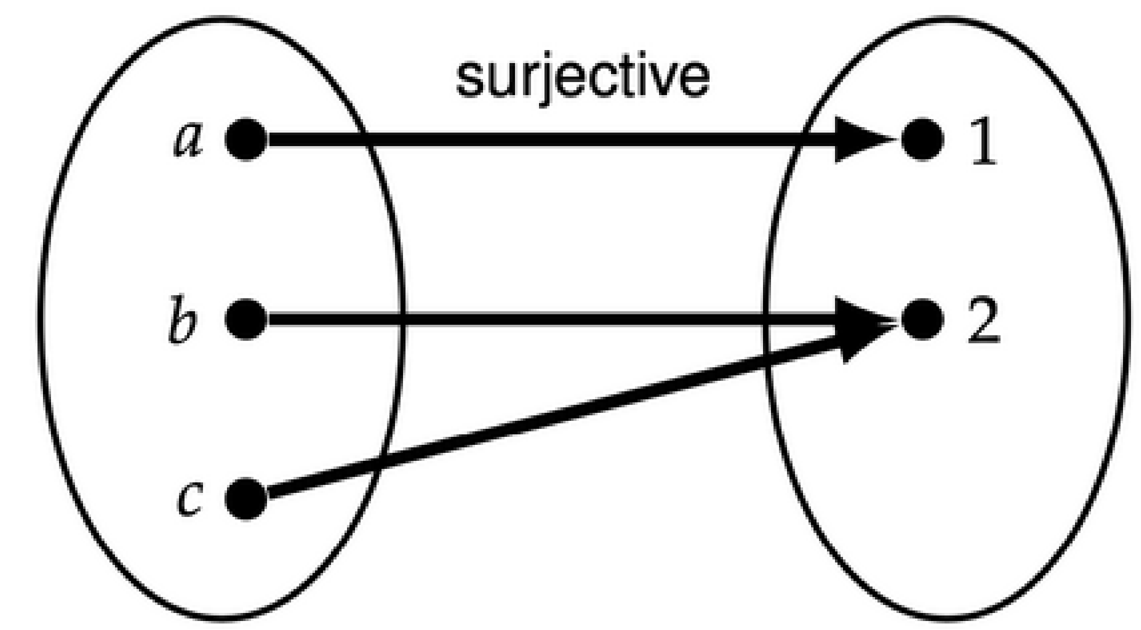
at most one element from domain maps to an element of the codomain

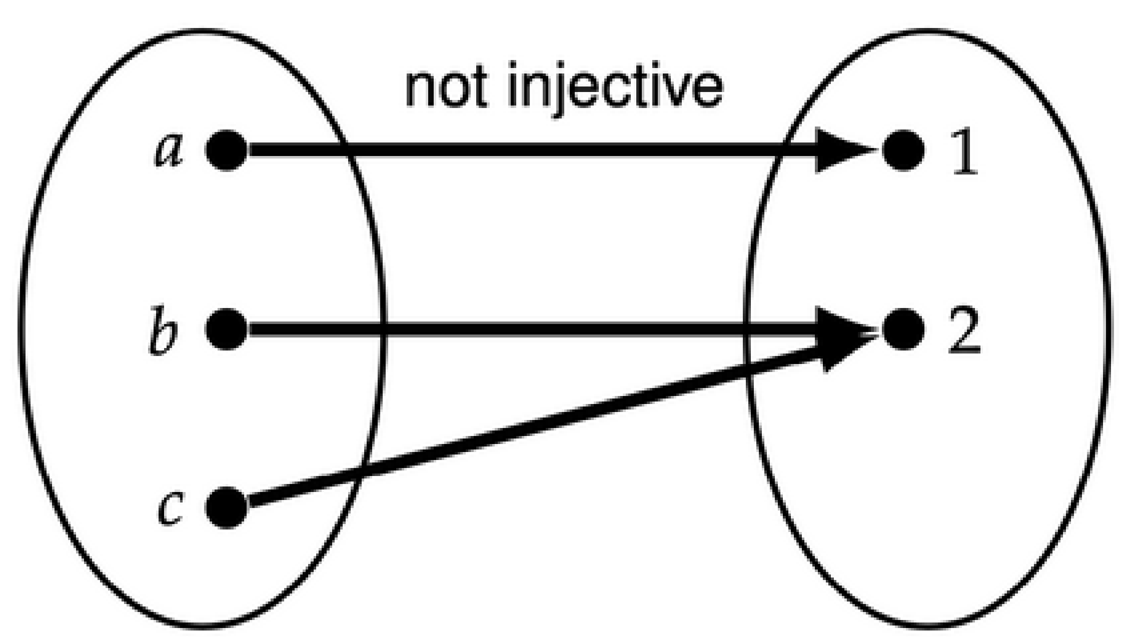
range = codomain

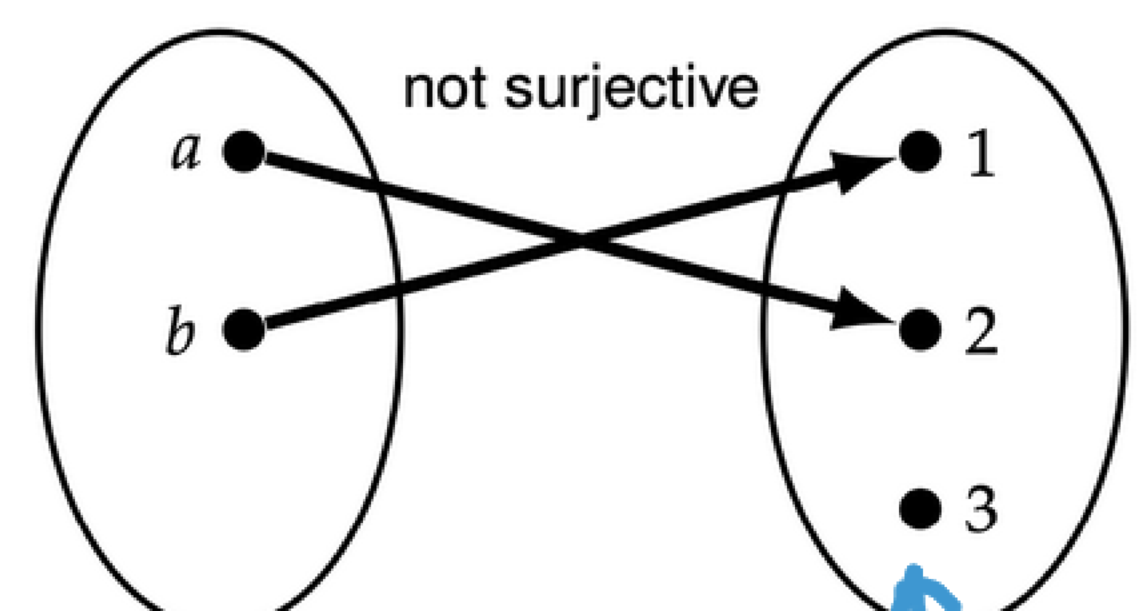
Injective: *one-to-one*

Surjective: *onto*







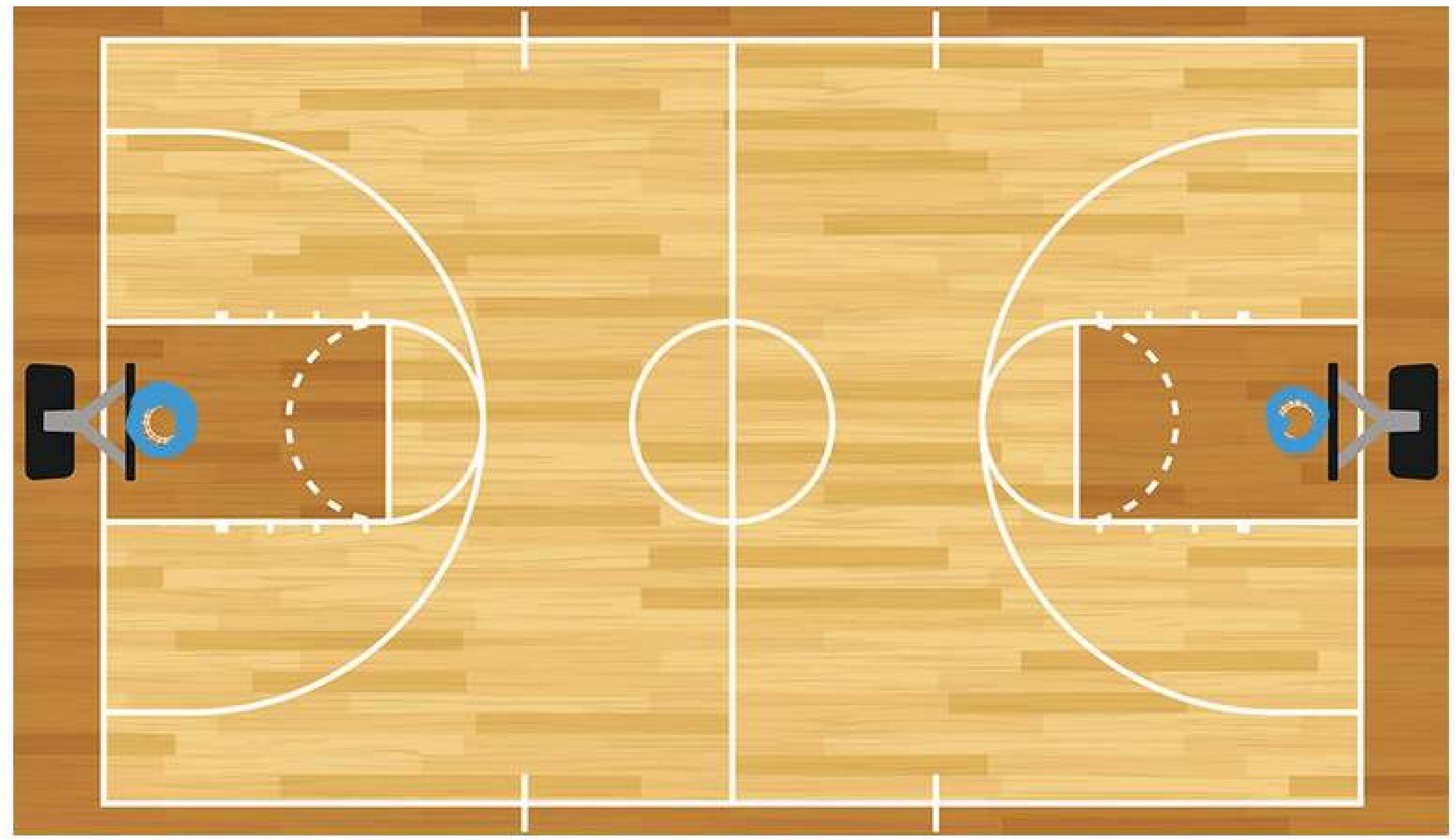


left out

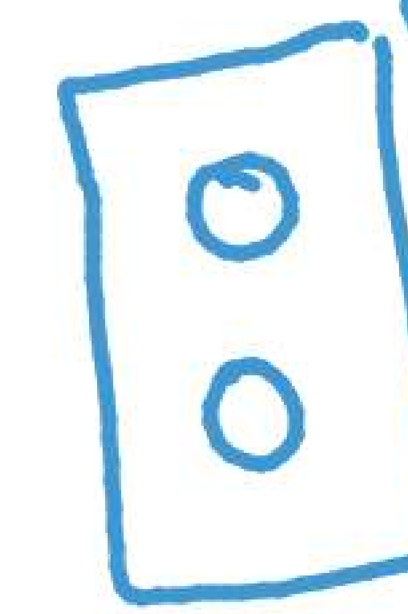
injective + surjective = bijective

Consider the following basketball situations. Let the domain be the players and let the codomain (always) be the two nets.

injective? → 1-to-1
surjective? codomain = range
bijective?



codomain (2 nets)



1. 1-on-1 half-court game. not injective, not surjective (1 net left out)
2. 5-on-5 full-court game. not injective, surjective
3. 1-on-1 full-court game. injective, surjective → bijective
4. Philip scores on the opposing net but then accidentally scores on his own net seconds later. not a function

Exercise 2: Determine if the following functions are injective, surjective or bijective.

$f: \mathbb{Z} \to \mathbb{Z}$

injective: 1-to-1

surjective: codomain = range

1. $f(n) = n - 1$ injective
surjective → bijective

2. $f(n) = n^2 + 1$ not injective (-4, 4 map to same value)
not surjective (no preimage for negative numbers)

3. $f(n) = n^3$ injective
not surjective (2 does not have a preimage)

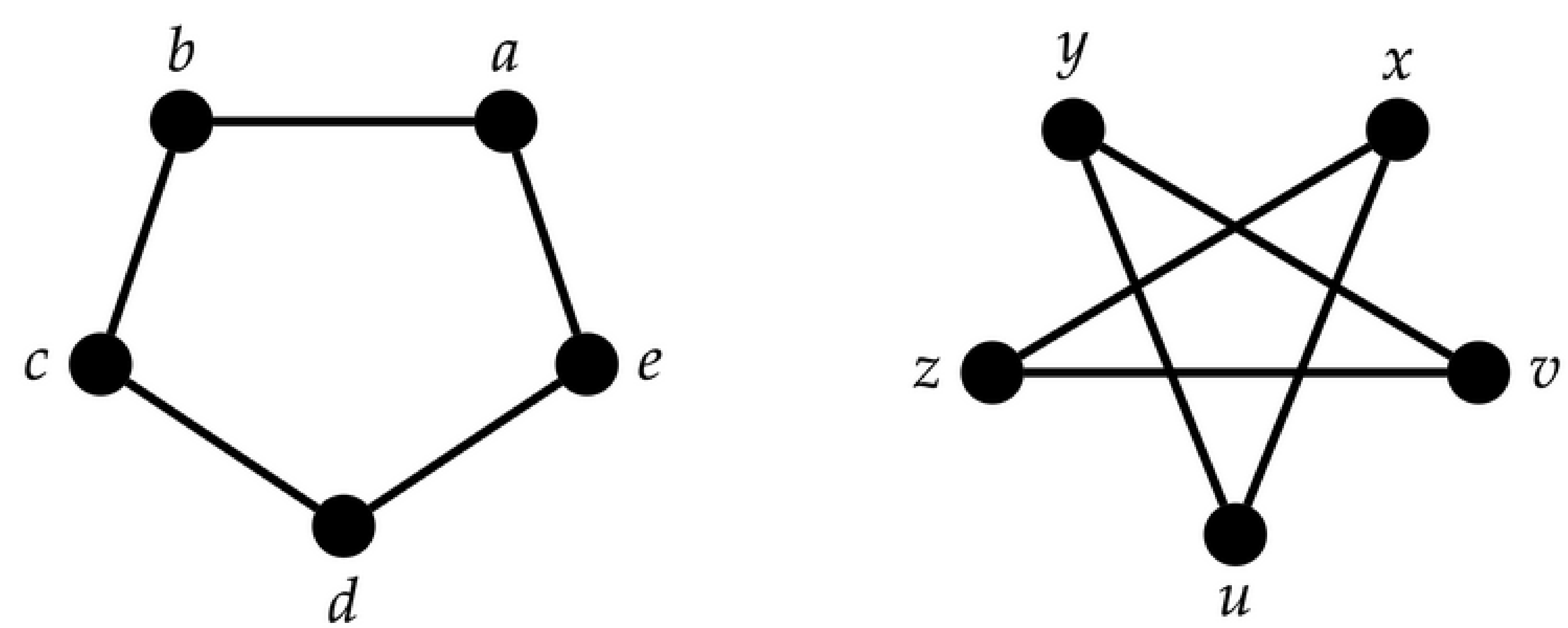
Application to graph theory: isomorphism.

Definition: an *isomorphism* between two graphs G and H is a bijection $f: V(G) \to V(H)$ such that

relabeling of vertices

$$(u, v) \in E(G) \iff (f(u), f(v)) \in E(H)$$

In words: every edge in first graph G can be found in the second grap H with relabeled vertices.



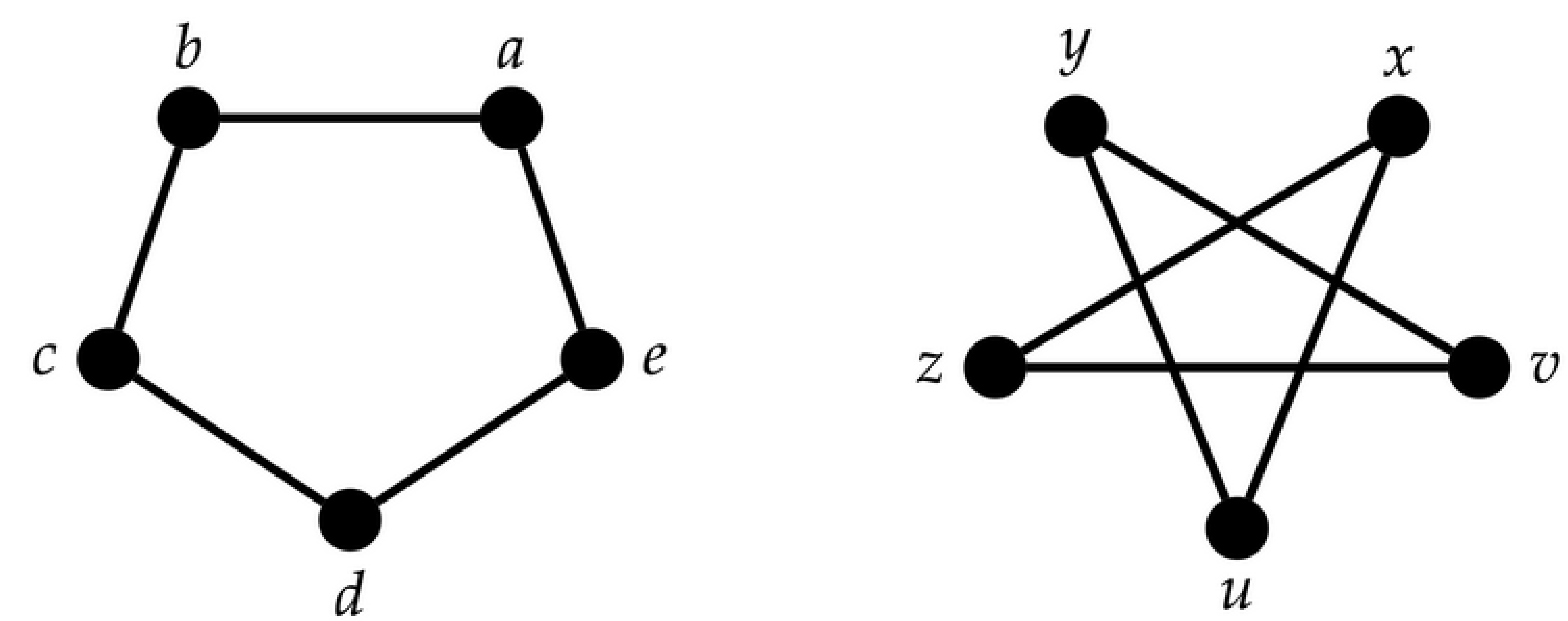
yes

Are these graphs isomorphic? If so, what is the relabeling?

$f(a) = x \quad f(b) = z \quad f(c) = v \quad f(d) = y \quad f(e) = u$

Testing for an isomorphism.

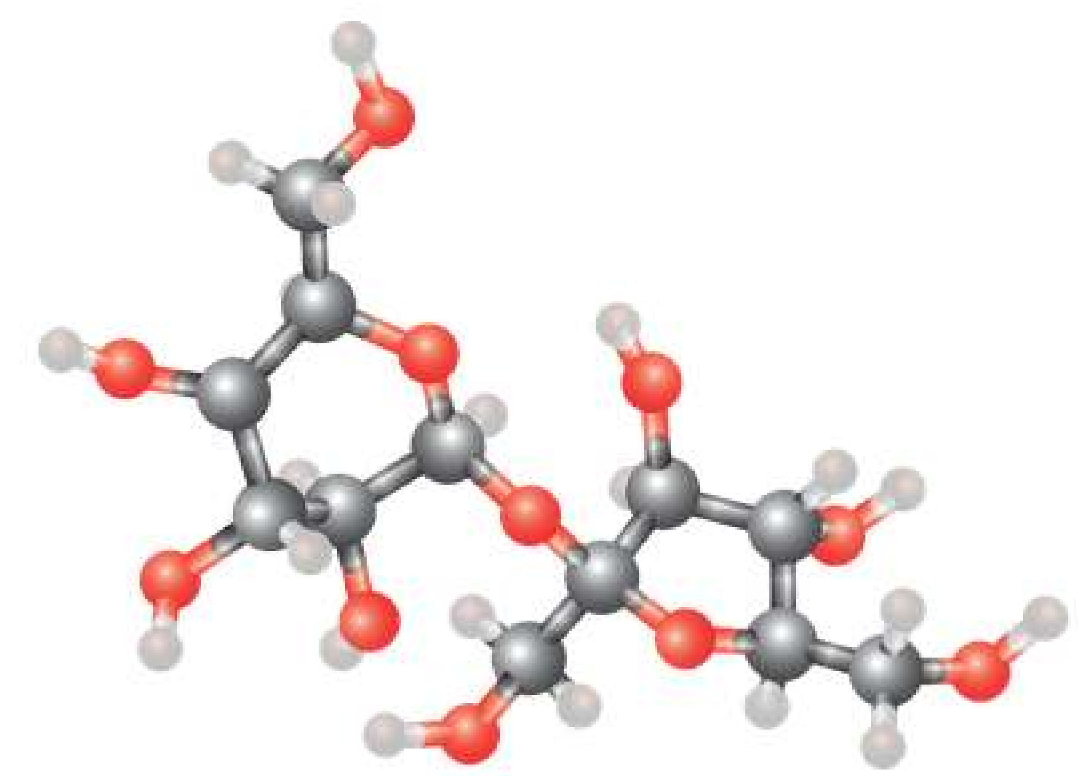
1. Check if the number of vertices and edges are the same.
2. Check all vertex degrees in G are found in H.
3. Check cycle lengths in G are found in H.



Isomorphism is an equivalence relation
(all isomorphic graphs are in the same equivalence class)

Applications of isomorphisms.

Chemistry: *does this chemical compound exist already?*



Circuits: *is this design protected by intellectual property?*