Where to Get Eclipse Glasses

Students, Faculty, and Staff can obtain eclipse glasses at the following times:

Friday 4/5, 12:00 - 1:30pm
Axinn Winter Garden

Sunday 4/7, 1:00 - 4:00pm
McCardell Bicentennial Hall, Great Hall

Monday 4/8, 1:30 - 4:30pm
Battell Beach Viewing Location

Midd ID is required to pick up eclipse glasses. Please follow all instructions printed on the eclipse glasses.
Rule #1 (Mapping Rule): relating the cardinalities of two sets.

Injective: *one-to-one*

\[ |A| \leq |B| \]

Surjective: *onto*

\[ |A| \geq |B| \]

**bijective:** injective and surjective: \( |A| \leq |B| \land |A| \geq |B| \rightarrow |A| = |B| \)

Main idea: 1) if there is a bijective function between two sets
2) count the set we know how to count, infer the size of other set.
Rule #2: Product rule.

- Event $A$ can occur in $m$ ways.
- Event $B$ can occur in $n$ ways.
- Total number of possible events: $m \times n$.

Example: Number of possible Vermont license plates?
(3 letters and then 3 numbers)

\[
\begin{align*}
a-z & \quad a-z & \quad a-z & \quad 0-9 & \quad 0-9 & \quad 0-9 \\
26 \times 26 \times 26 \times 10 \times 10 \times 10 \\
= 26^3 \times 10^3
\end{align*}
\]
Rule #3: Addition rule.

- Event $A$ can occur in $m$ ways.
- Event $B$ can occur in $n$ ways.
- Assume $A$ and $B$ are disjoint.
- $A$ or $B$ occurs in $m + n$ ways.

Example: Number of possible passwords? Assume passwords must:
- contain 6-8 characters,
- each character is either an uppercase letter or a number,
- must contain at least one number.

$$P_n = \# \text{ passwords of length } n \rightarrow P_6 + P_7 + P_8$$

eg. $n=6$

$$26 + 26 + 26 \cdot 26 = 26^6$$

we need 1 number what would violate this rule?

all letters

26 26 26 26 26 26 = 26^6

$$(36^6 - 26^6) + (36^7 - 26^7) + (36^8 - 26^8)$$

final result
Rule #4: Principle of Inclusion-Exclusion (PIE).

- Event $A$ can occur in $|A|$ ways.
- Event $B$ can occur in $|B|$ ways.
- Assume $A$ and $B$ are not disjoint.
- $A$ or $B$ occurs in $|A \cup B| = |A| + |B| - |A \cap B|$ ways.

Example: How many 5-bit strings start with 1 or end with 00?

$|A| = 2^4$
$|B| = 2^3$
$|A \cap B| = 2^2$

# bit strings $= 2^4 + 2^3 - 2^2 = 20$
The Pigeonhole Principle:

In general: If \( n \) objects are placed into \( k \) boxes, then there is at least one box which contains \( \geq \left\lceil \frac{n}{k} \right\rceil \) objects.  

E.g., \( \left\lceil \frac{3}{2} \right\rceil = \left\lceil 1.5 \right\rceil = 2 \)

Example: Birth month question from beginning of class.

\( n = 22 \)
\( k = 12 \)
\( \left\lceil \frac{22}{12} \right\rceil = 2 \)
Exercise 1: How many surjective functions are there from a set $A$ to a set $B$ if $|A| = n$ and $|B| = 2$?

Steps: (show your work 😊)
- How many functions in total? $2^n$
- How many not surjective? (all $a \in A$ map to $b_1$, all $a \in A$ map to $b_2$) 2 functions

# Surjective functions $= 2^n - 2$.  

1) map every $a \in A$
2) don't exclude any $b \in B$
Exercise 2: In how many ways can a photographer at a wedding arrange six people in a row, including the married couple, if:

(a) the married couple is next to each other?

\[
2 \times 5! = 2 \times 120 = 240
\]

(b) the married couple is not next to each other?

\[
\text{Total ways} - \text{ways in which they are next to each other}
\]

\[
= 6! - 2 \times 5!
\]

\[
= 720 - 240 = 480
\]