go/eclipse24

Where to Get Eclipse Glasses

Students, Faculty, and Staff can obtain eclipse glasses at the following times:

Friday 4/5, 12:00 - 1:30pm

Axinn Winter Garden

Sunday 4/7, 1:00 - 4:00pm

McCardell Bicentennial Hall, Great Hall

Monday 4/8, 1:30 - 4:30pm

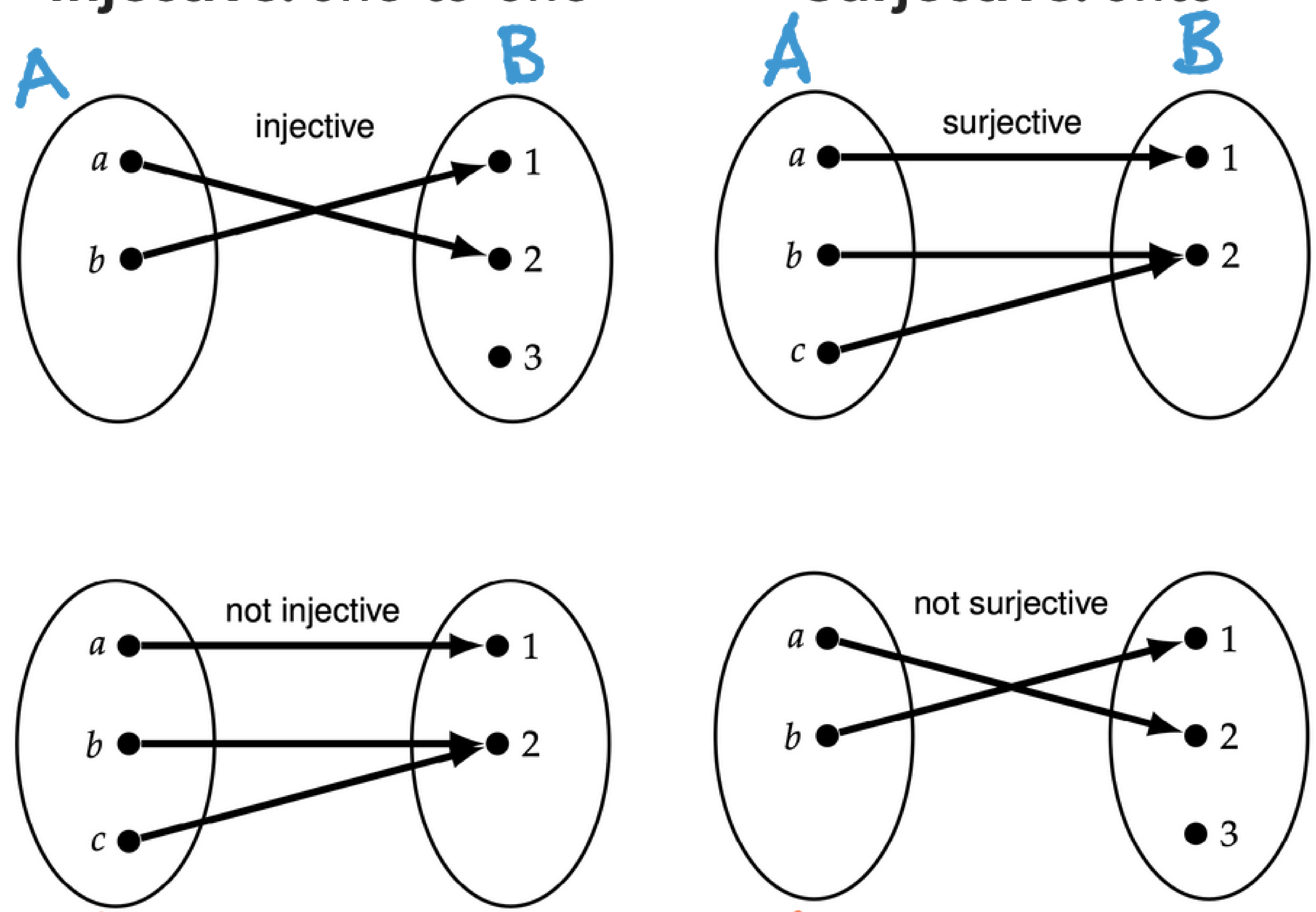
Battell Beach Viewing Location

Midd ID is required to pick up eclipse glasses. *Please follow all instructions printed on the eclipse glasses.*

Rule #1 (Mapping Rule): relating the cardinalities of two sets.

Injective: *one-to-one* **Surjective:** *onto*

$|A| \leq |B|$



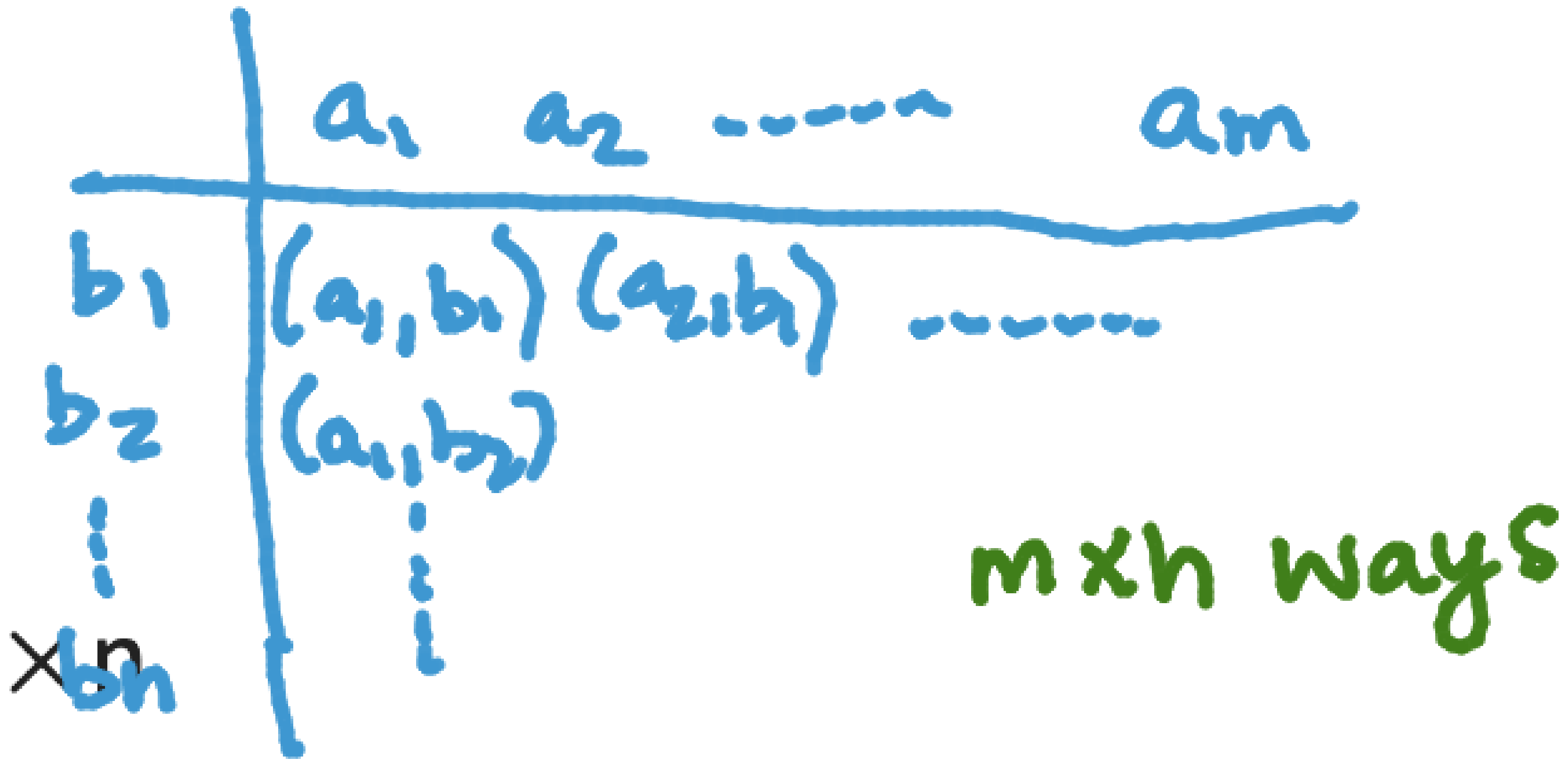
$|A| \geq |B|$

bijective: injective and surjective: $|A| \leq |B| \wedge |A| \geq |B| \rightarrow |A| = |B|$

main idea: 1) if there is a bijective function between two sets
2.) count the set we know how to count, infer the size of other set.

Rule #2: Product rule.

- Event A can occur in m ways.
- Event B can occur in n ways.
- Total number of possible events: $m \times n$



Example: Number of possible Vermont license plates?
(3 letters and then 3 numbers)

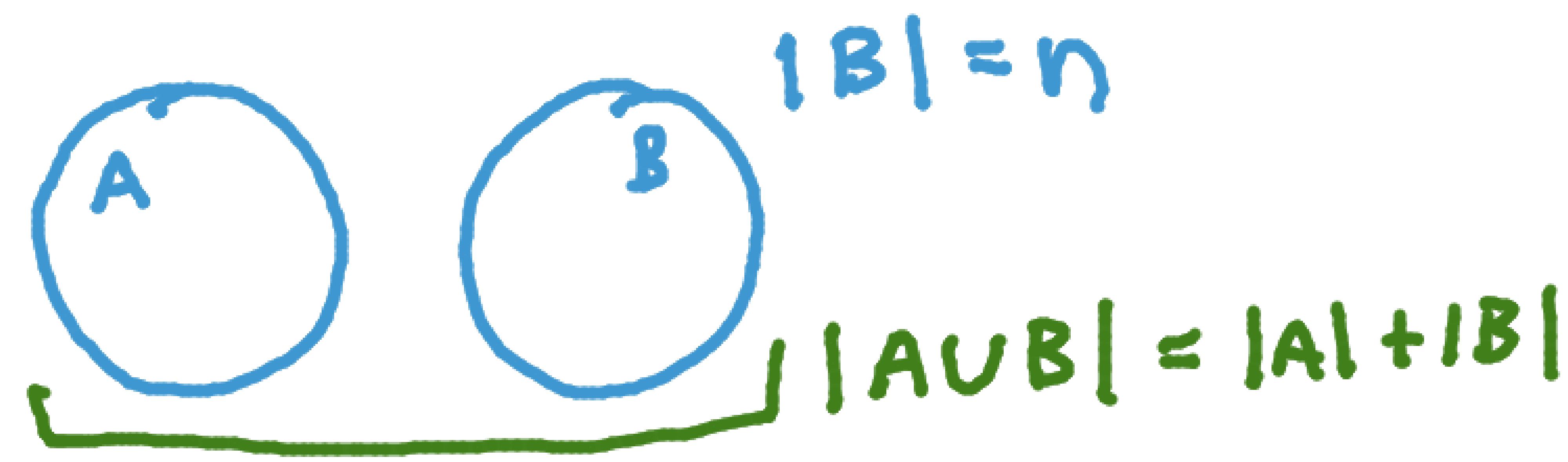
a-z a-z a-z 0-9 0-9 0-9

$26 \times 26 \times 26 \times 10 \times 10 \times 10$

$= 26^3 10^3$

Rule #3: Addition rule.

- Event A can occur in m ways.
- Event B can occur in n ways.
- Assume A and B are **disjoint.**
- A or B occurs in $m + n$ ways.



Example: Number of possible passwords? Assume passwords must:

- contain 6-8 characters,
- each character is either an uppercase letter or a number,
- must contain at least one number.

passwords len 6
passwords len 7
passwords len 8

P_n = # passwords of length n → $P_6 + P_7 + P_8$

e.g. n=6 _ _ _ _ _ _ = 36^n → #passwords (len n) = $36^n - 26^n$

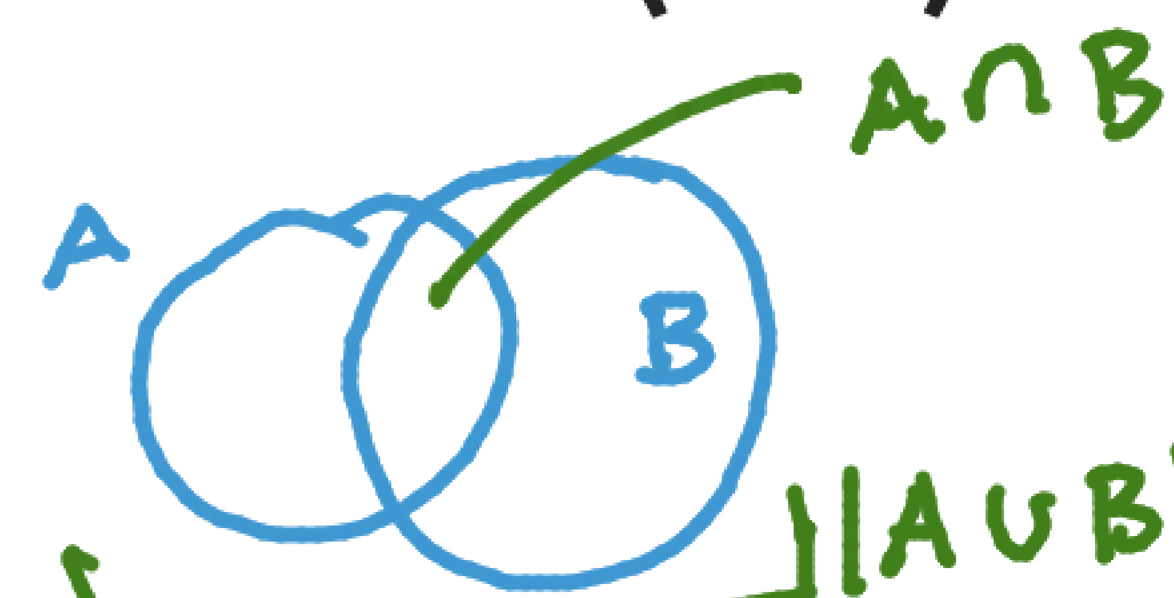
a-z or 0-9 (26+10) = 36

we need 1 number
what would violate this rule?
all letters
26 26 26 26 26 26 = 26^n

final result
$(36^6 - 26^6) + (36^7 - 26^7) + (36^8 - 26^8)$

Rule #4: Principle of Inclusion-Exclusion (PIE).

- Event A can occur in $|A|$ ways.
- Event B can occur in $|B|$ ways.
- Assume A and B are **not disjoint**.
- A or B occurs in $|A \cup B| = |A| + |B| - |A \cap B|$ ways.



$|A \cup B| = |A| + |B| - |A \cap B|$

Example: How many 5-bit strings start with 1 or end with 00?

1 x x x x (A) or x x x 0 0 (B)

1 x x 0 0 $|A \cap B|$

$|A| = 2^4$

$|B| = 2^3$

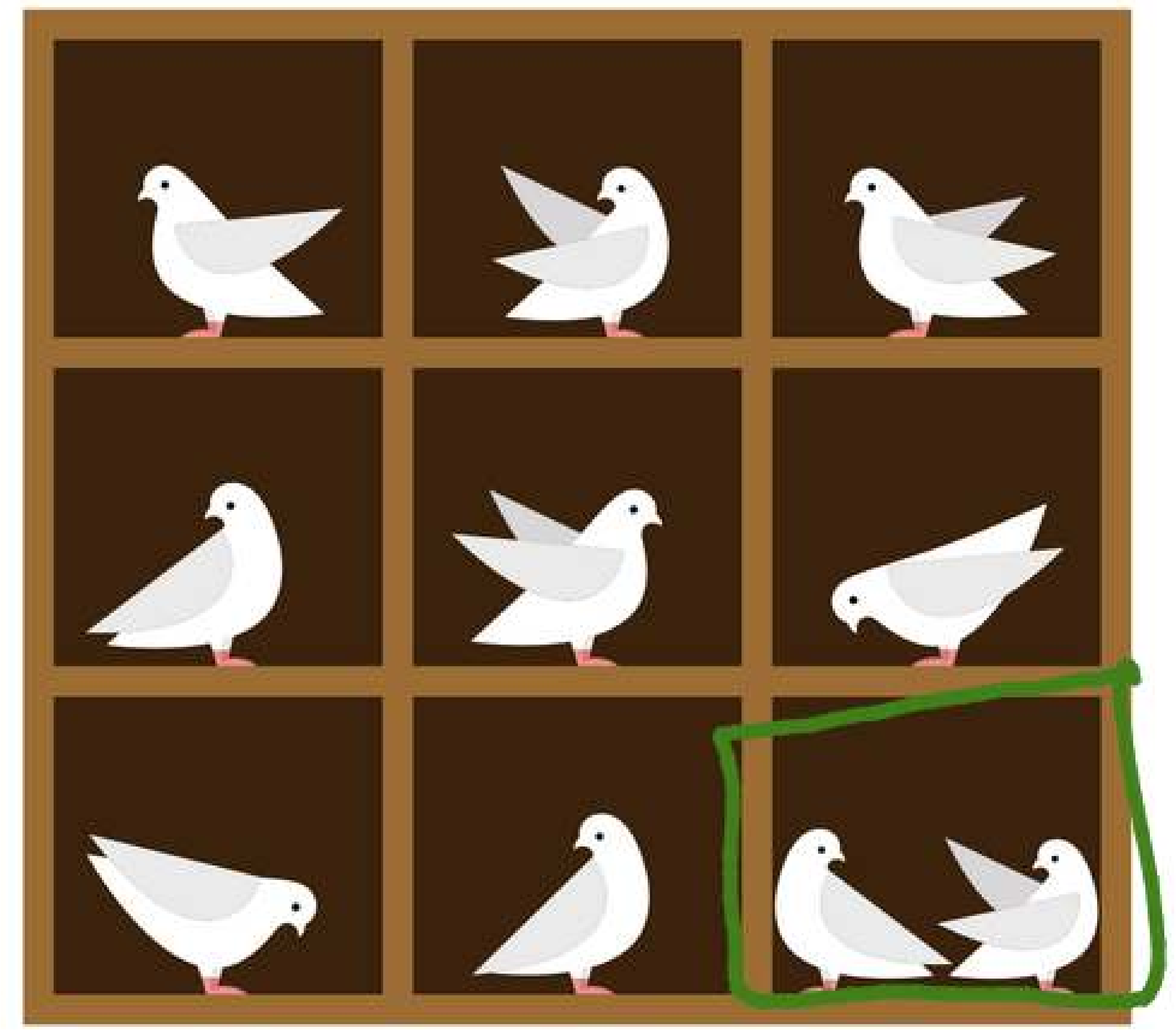
$|A \cap B| = 2^2$

$\#\text{ bit strings} = 2^4 + 2^3 - 2^2$

$= 20$

The Pigeonhole Principle:

place n+1 pigeons into n boxes.



one box must have 2 pigeons

In general: If n objects are placed into k boxes, then there is at least one box which contains $\geq \lceil \frac{n}{k} \rceil$ objects.

e.g. $\lceil \frac{7}{2} \rceil = \lceil 3.5 \rceil = 4$

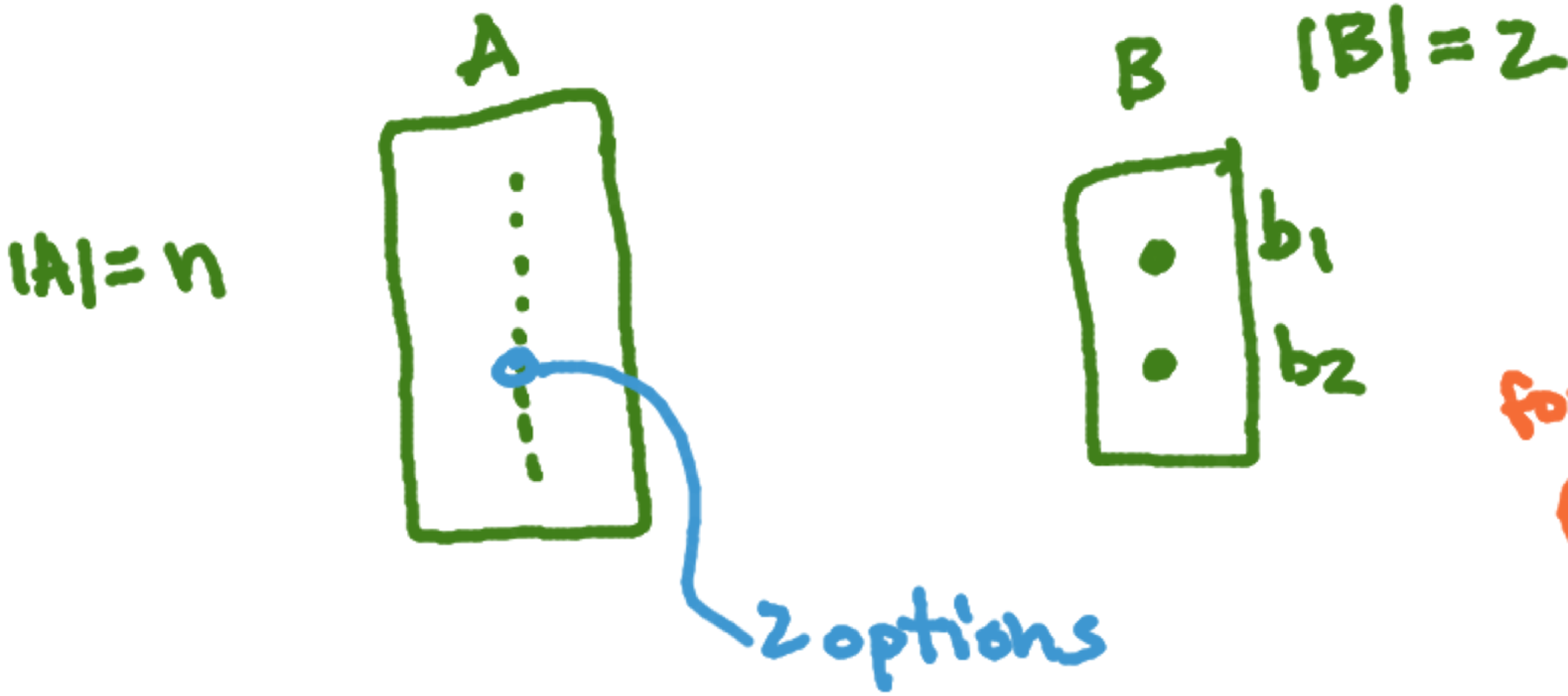
Example: Birth month question from beginning of class.

$n = 22$

$k = 12$

$\lceil \frac{22}{12} \rceil = 2$

Exercise 1: How many surjective functions are there from a set A to a set B if $|A| = n$ and $|B| = 2$?



remember: 1) map every $a \in A$

for "onto" function → 2) don't exclude any $b \in B$

steps: (show your work :))

how many functions in total? 2^n

how many not surjective? all $a \in A$ map to b_1, all $a \in A$ map to b_2] 2 functions

surjective functions $= 2^n - 2$.

Exercise 2: In how many ways can a photographer at a wedding arrange six people in a row, including the married couple, if:

- (a) the married couple is next to each other?

$\boxed{P_1\ P_2}$ $\underline{P_3}$ $\underline{P_4}$ $\underline{P_5}$ $\underline{P_6}$

$\boxed{P_1\ P_2}$ $\boxed{P_2\ P_1}$

$\underline{5}$ $\underline{4}$ $\underline{3}$ $\underline{2}$ $\underline{1}$

$2 \times 5!$

$= 240$

- (b) the married couple is not next to each other?

total # ways − # ways in which they are next to each other

$= 6! - 2\times 5!$

$= 720 - 240 = \boxed{480}$