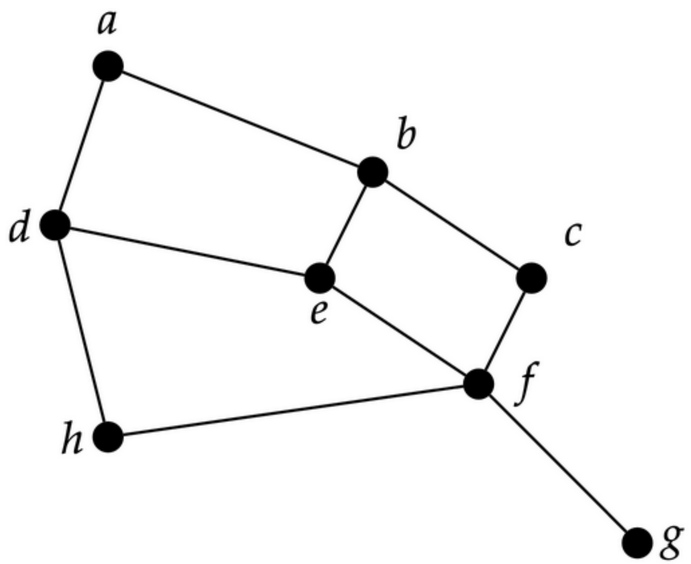
Walks.

Definition 1: A walk (of length k) in a graph $G = (V, E)$ is a sequence of vertices that are connected by k edges.

Definition 2: A closed walk is a walk which starts and ends at the same vertex.



Which of the following are true? (slido.com #1868080) 26

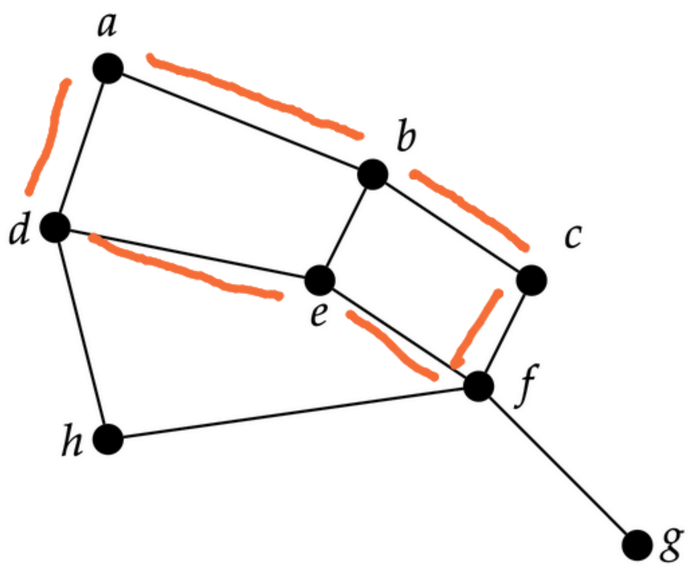
- [x] c - b - a - d - h - f - e is a walk of length 6
- [ ] c - b - a - d - h - f - e is a walk of length 7
- [ ] d - e - c - f - e is a closed walk of length 5
- [ ] d - e - c - f - e is a closed walk of length 6
- [x] d - e - b - c - f - g - f - h - d is a closed walk of length 8
- [ ] d - e - b - c - f - g - f - h - d is a closed walk of length 9

Voting as Anonymous Send

Paths and Cycles.

Definition 3: A _path_ is a _walk_ where all the vertices are _different_.

Definition 4: A _cycle_ is a _closed_ walk of (length _> 2_) in which all the vertices are _different_.



Which of the following are true? (slido.com #1868080) 32

- [x] a - d - e - b - c is a path
- [ ] a - d - e - b - e - f is a path
- [x] a - b - c - f - e - d - a is a cycle
- [x] e - f - h - d - e is a cycle

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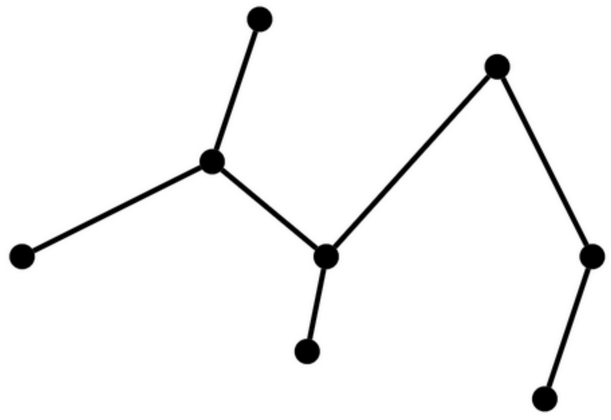
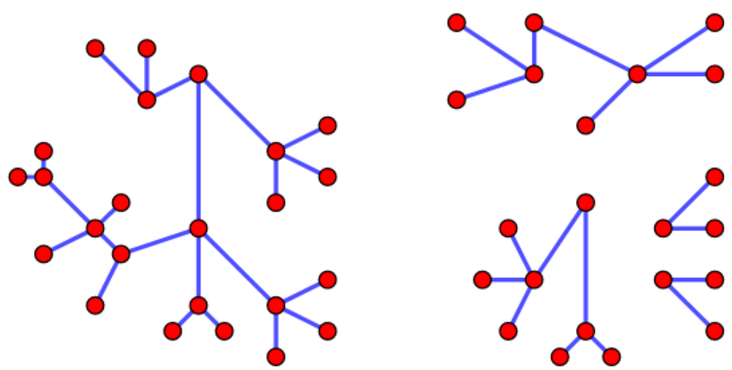
Send

Definition 5: Vertices u and v are said to be _connected_ if there is a _path_ between u and v.

Finally! A tree is a *connected* graph with no cycles (*acyclic*).



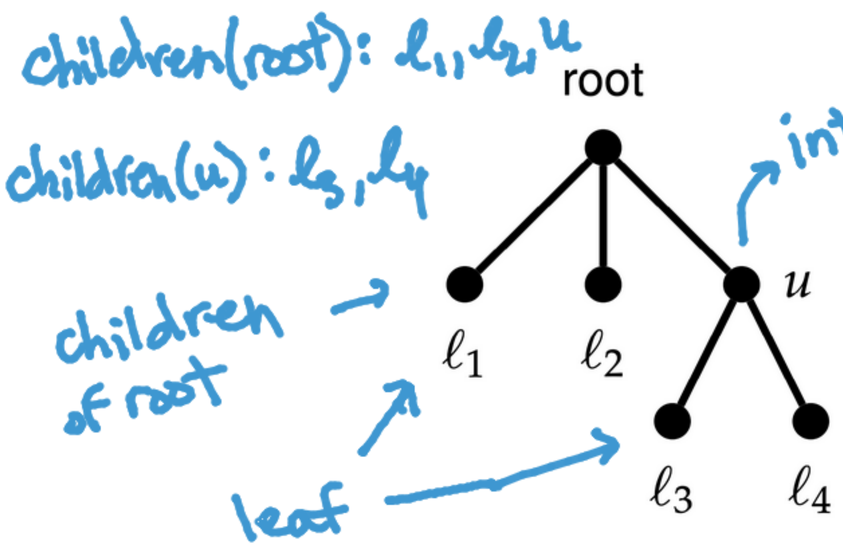
No cycles but not connected? → **Forest.**



Tree Forest

Rooted and k-ary trees.

Definition 6: A rooted tree is a tree in which a single vertex is designated as the root and every edge is directed away from the root.



What is k for this tree? (slido.com #1868080) 31

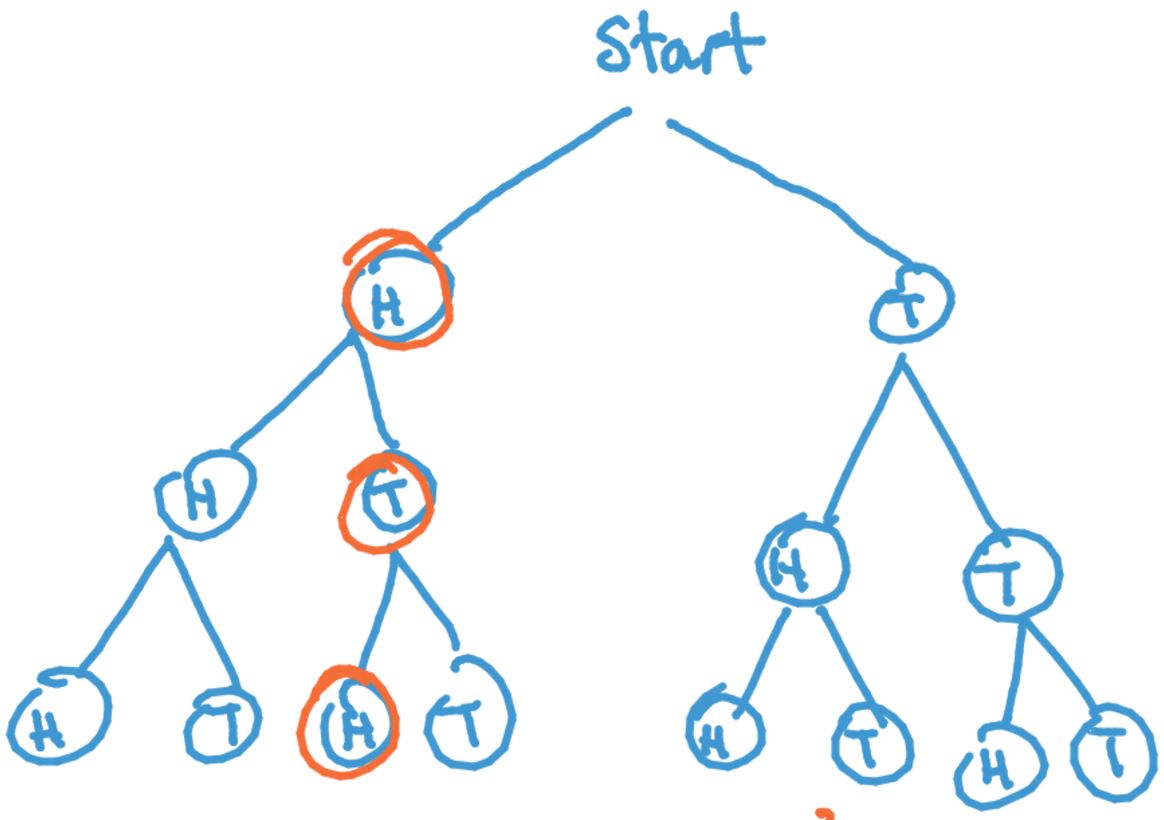
- 2
- 3
- 5
- 6

Voting as Anonymous Send

k = 2
binary tree

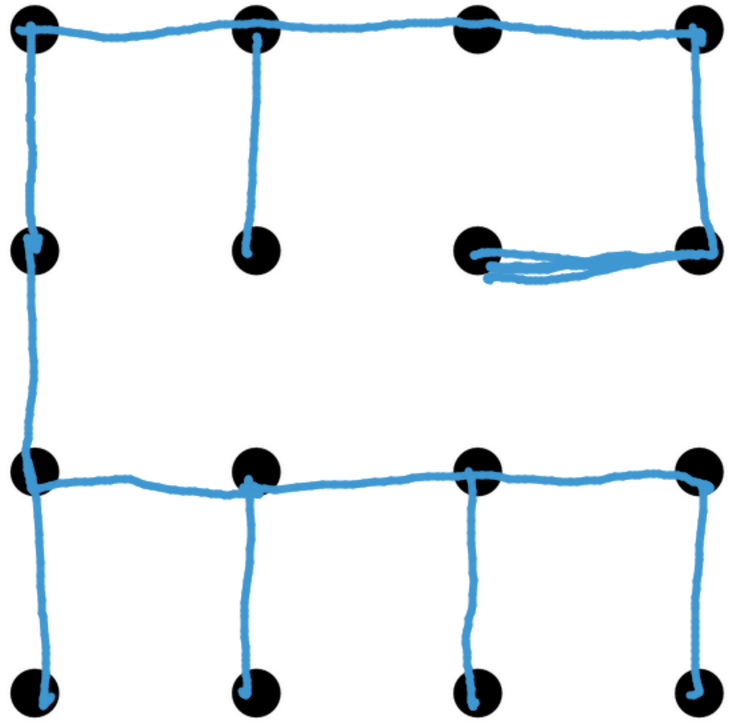
Definition 7: A k-ary is a tree in which each node has $\leq k$ children.

Exercise 1: draw the binary tree resulting from all possible outcomes of flipping a coin three times.



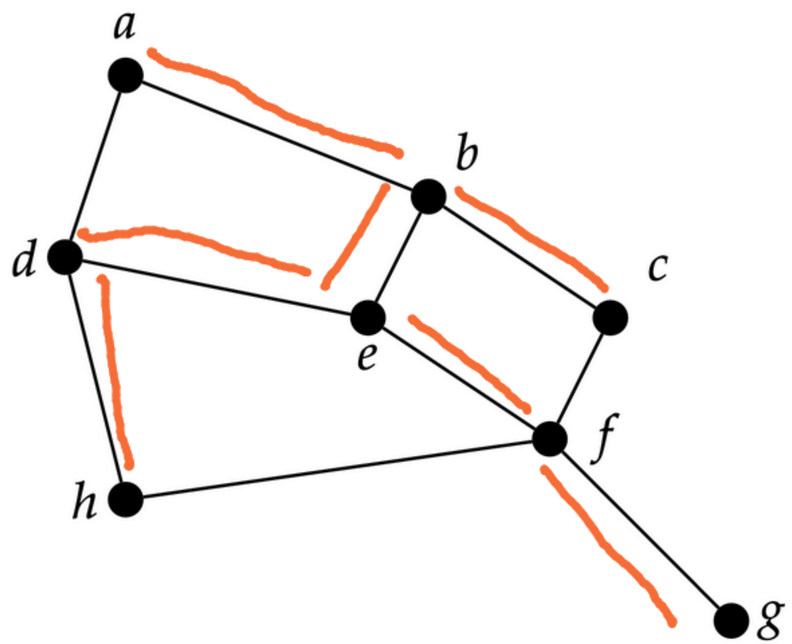
1. How many leaves are there? 8 2^3
2. What is the probability of getting the sequence H-T-H (3 flips)? $\frac{1}{8}$
3. What is the probability of getting the sequence H-T-T-T-H-H (6 flips)? $\frac{1}{2^6}$

Exercise 2: make a tree that has all of these vertices!



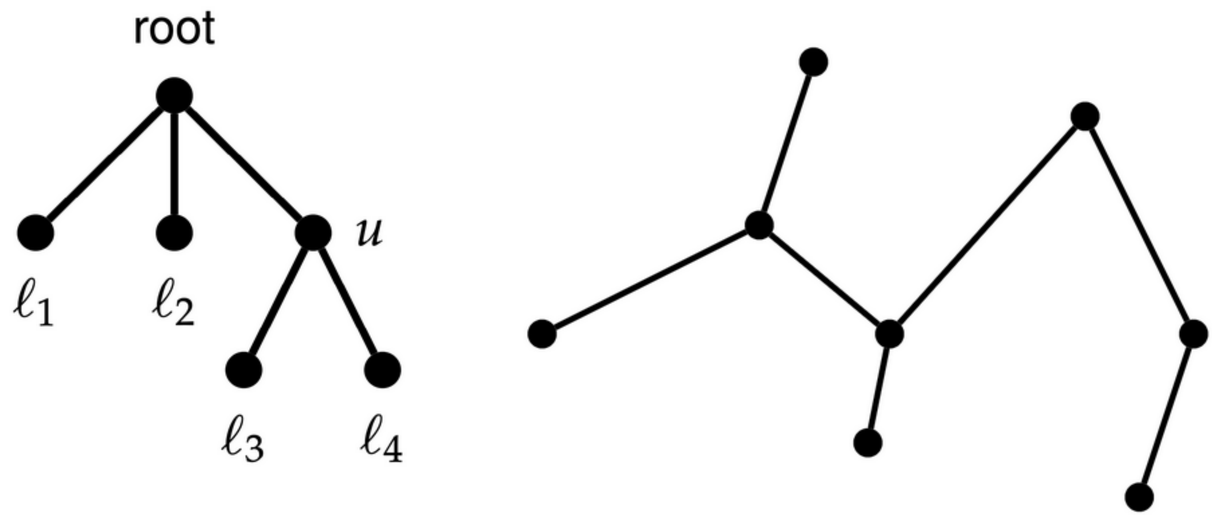
Definition 8: A Spanning tree of a connected graph $G = (V, E)$ is a subgraph of G with the same vertices as G.

Exercise 3: make a spanning tree of this graph.



1. Is your spanning tree unique? not unique
2. How many edges are there in your tree? 7
3. How many edges in the trees of your group members? Should also be 7

An important property of trees.



edges in a tree = # vertices − 1

proof: by induction on
- # vertices ?
- # edges ?

try one
if doesn't
work, try other