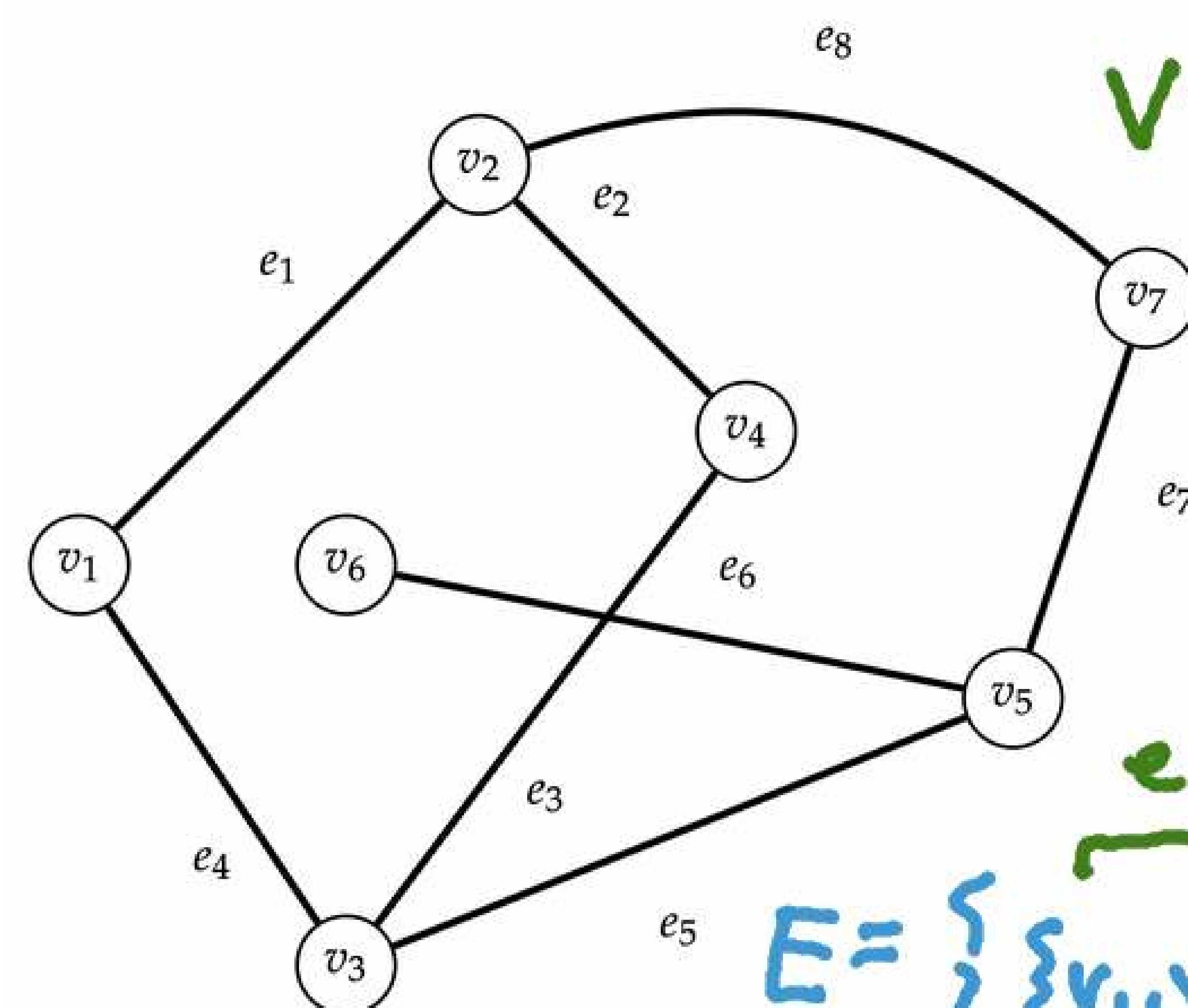
Your first graph!

Definition: A *graph* G is a pair of sets $G = (V, E)$ where V is a nonempty set of items called vertices (or nodes) and E is a set of 2-item subsets of V called edges.

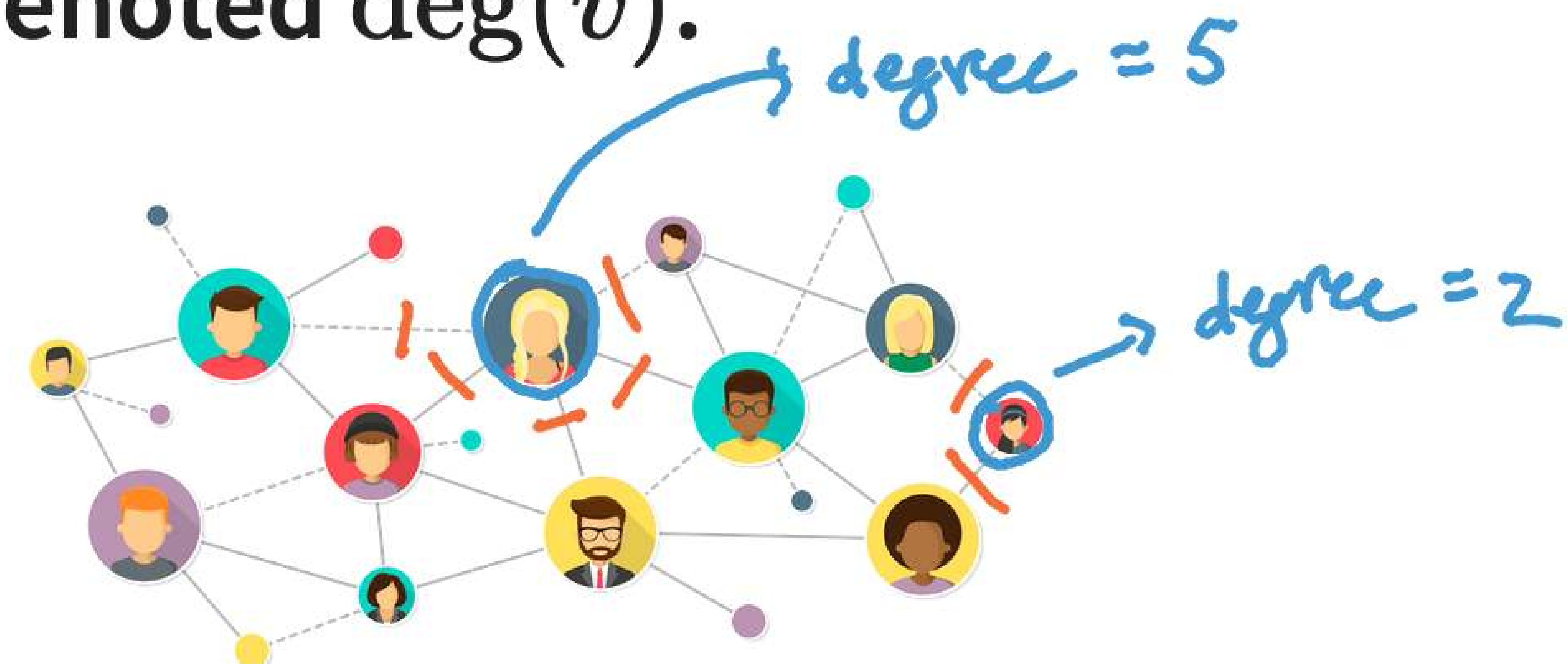


$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$

$E = \{ \overbrace{\{v_1, v_2\}}^{e_1}, \overbrace{\{v_2, v_4\}}^{e_2}, \overbrace{\{v_3, v_4\}}^{e_3}, \ldots \}$

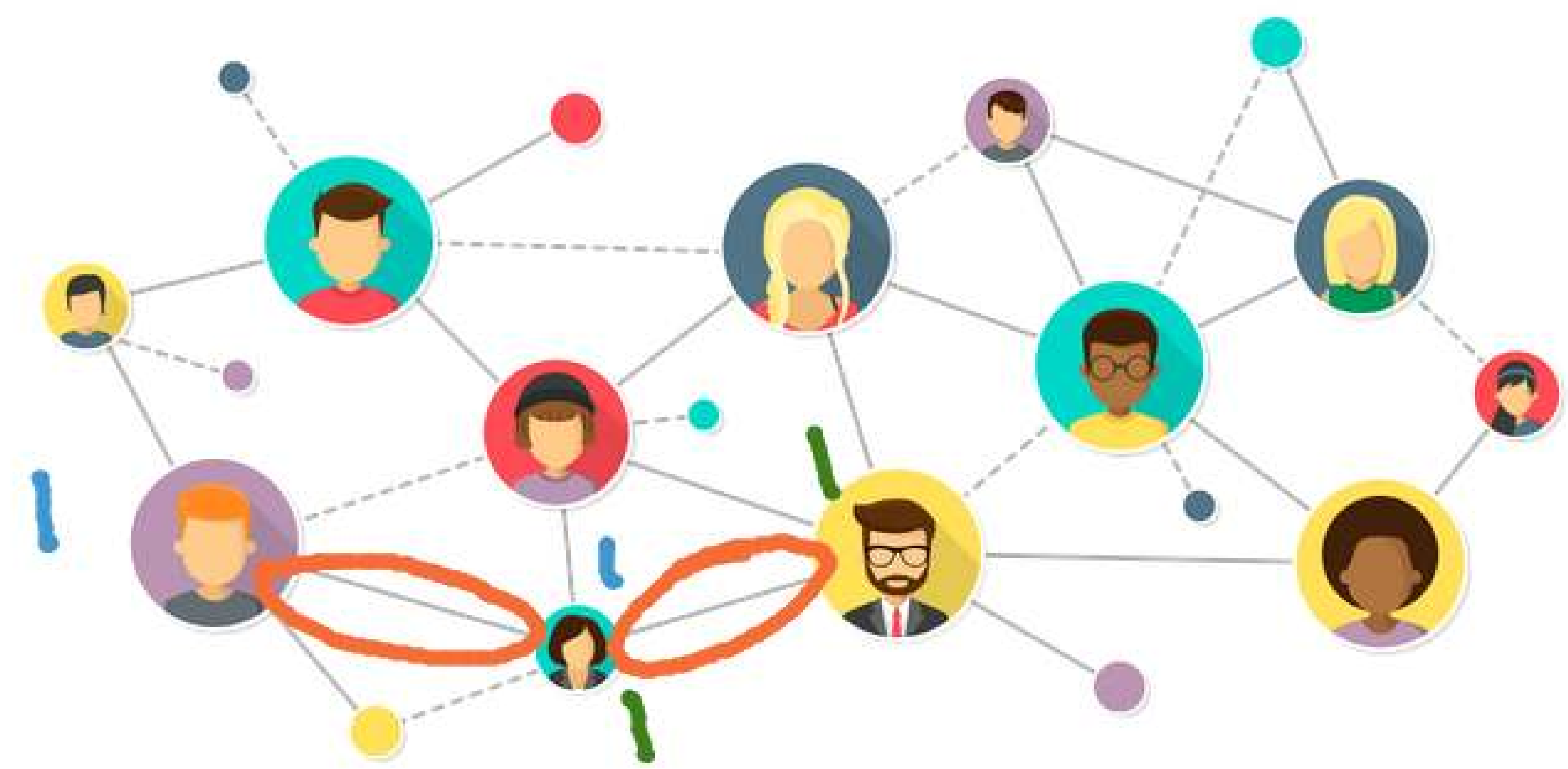
- two vertices $v_i,\ v_j$ are adjacent if $\{v_i, v_j\} \in E$
- the edge $e = \{v_i, v_j\}$ is incident to endpoints v_i and v_j.

The *degree* of a vertex v is the number of edges incident to v, denoted $\deg(v)$.



Puzzle: At a party with n people, each person p_i reported talking to m_i people ($0 \le m_i < n$). How many **total** conversations happened?

The handshaking lemma: the total sum of the vertex degrees is equal to twice the number of edges in a graph.



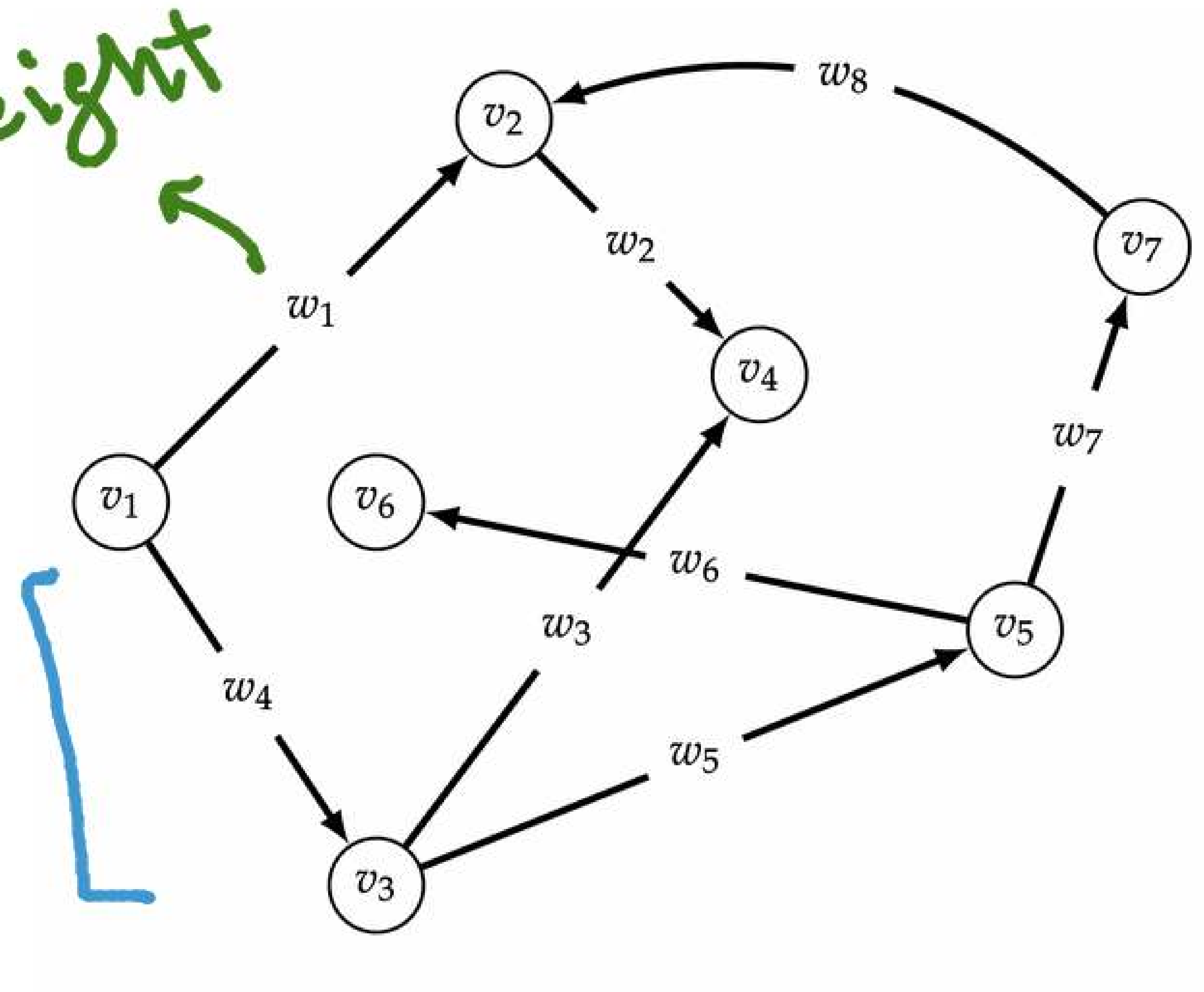
<u>proof:</u> We use a direct proof. Let $G=(V,E)$ be a graph. To count the sum of all vertex degrees, traverse every edge and add the contribution from each endpoint:

$$(1+1)+(1+1)+\ldots\ldots(\text{for every edge}) = 2|E| = \text{sum of vertex degrees.}$$

$\square$

Properties of graphs: weighted, simple, directed.

weight



(v_1, v_3)

directed edges

$v_1 \to v_3$

loop

multiedge

simple graphs means no loops, no multiedges

Is this graph simple? (slido.com: #1122662) 20

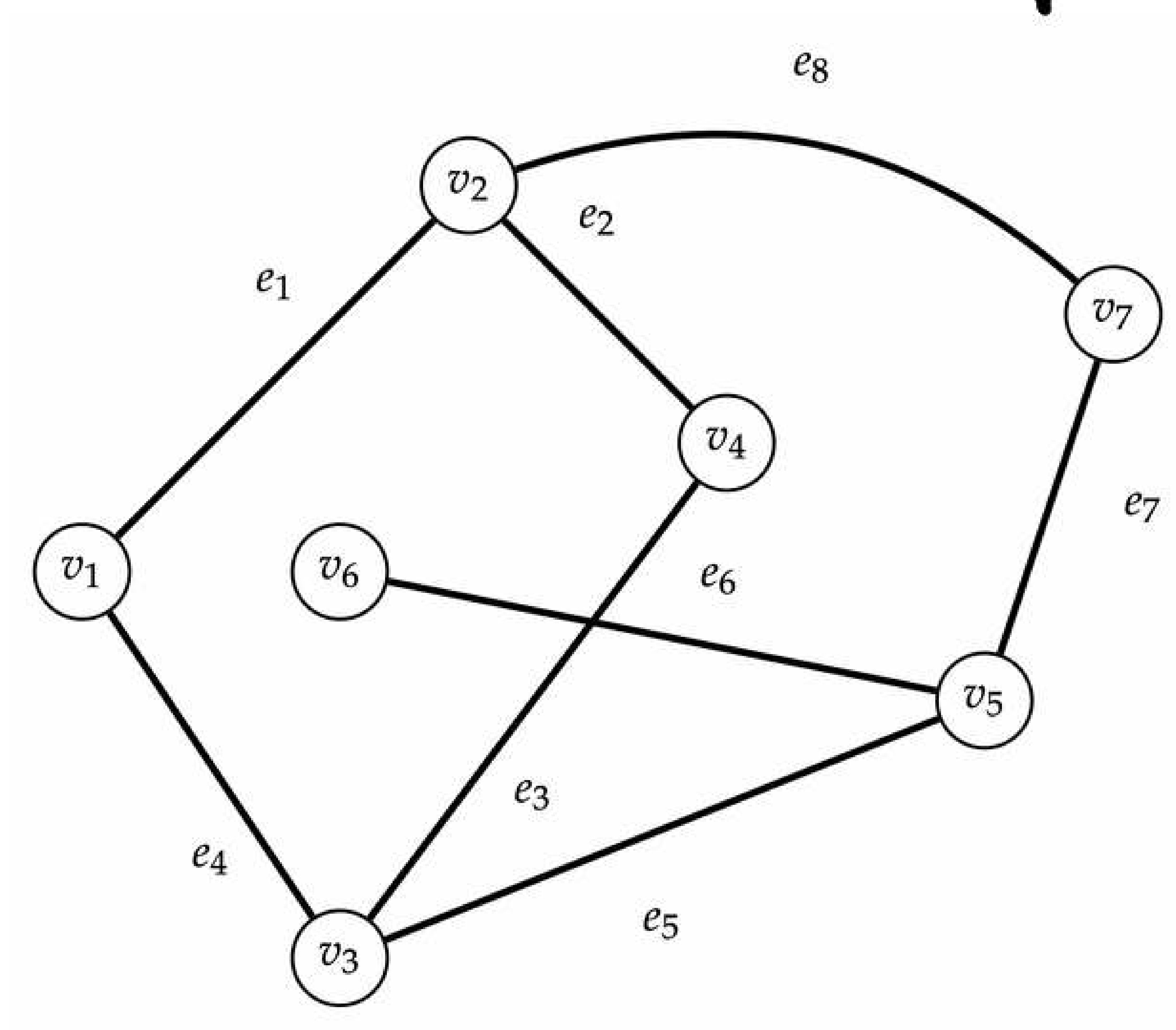
- Yes
- No

Voting as Anonymous

Send

Representing graphs with an *adjacency matrix*.

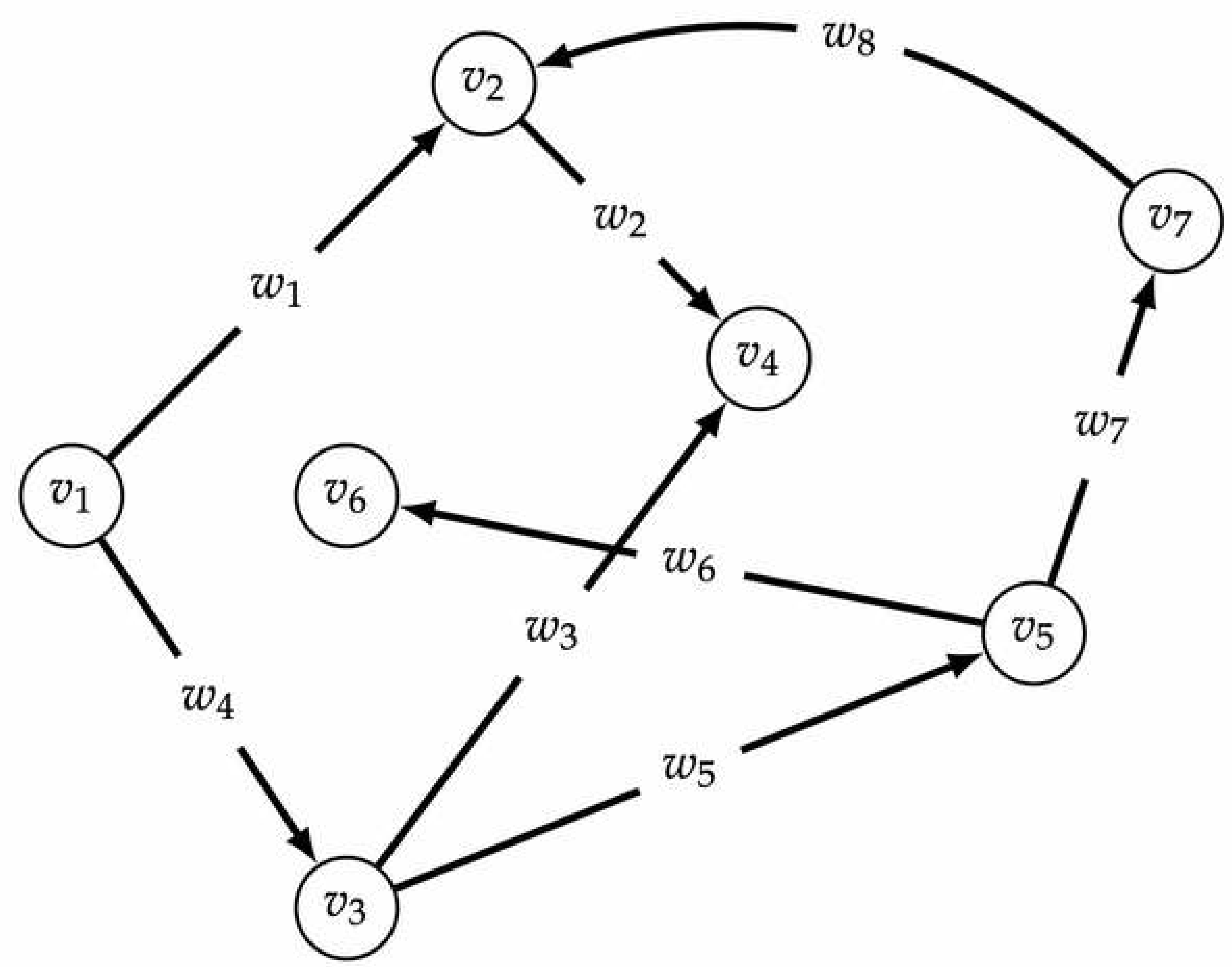
↳ put a 1 (or weight) if there is an edge between vertices



$$
\begin{array}{c c}
\begin{bmatrix}
0 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0
\end{bmatrix} &
\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \\
\begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} &
\end{array}
$$

0 otherwise

Representing graphs with *adjacency lists*.



vertex	list (adjacent to, weight)
v_1	(v_2, w_1), (v_3, w_4)
v_2	(v_4, w_2)
v_3	(v_4, w_3), (v_5, w_5)
v_4	—
v_5	(v_6, w_6), (v_7, w_7)
v_6	—
v_7	(v_2, w_8)