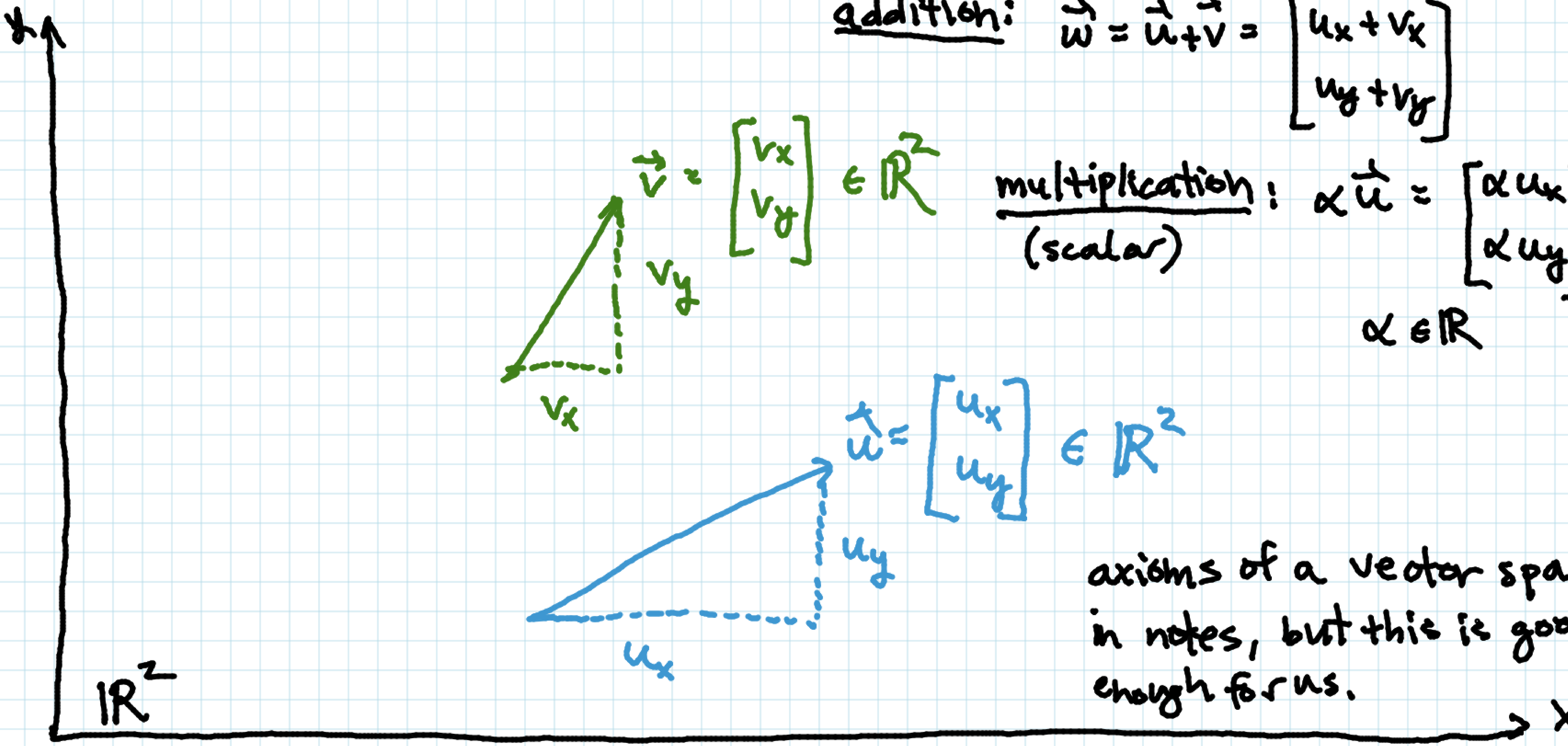
What is a vector? A "thing" with multiple components that satisfies certain properties.



$$\vec{v} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} \in \mathbb{R}^2$$

$$\vec{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} \in \mathbb{R}^2$$

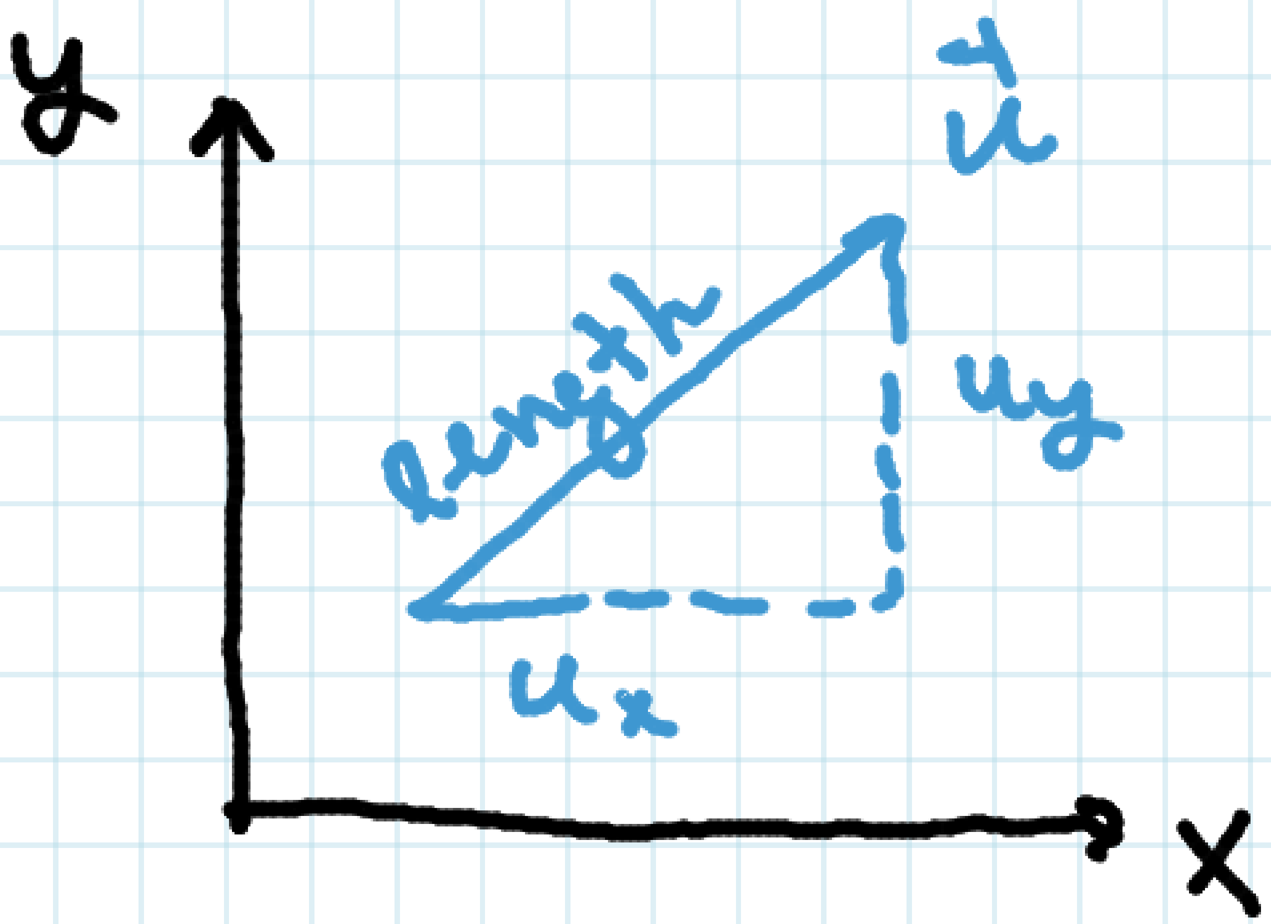
addition: $\vec{w} = \vec{u} + \vec{v} = \begin{bmatrix} u_x + v_x \\ u_y + v_y \end{bmatrix}$

multiplication (scalar): $\alpha \vec{u} = \begin{bmatrix} \alpha u_x \\ \alpha u_y \end{bmatrix}$

$\alpha \in \mathbb{R}$

axioms of a vector space in notes, but this is good enough for us.

Length of a vector (or "norm", or "magnitude").



$$\|\vec{u}\| = \sqrt{u_x^2 + u_y^2}$$

`\lVert \vec{u} \rVert`

e.g. calculate $\|\vec{u}\| = \sqrt{3^2 + 2^2 + 0^2 + 4^2} = \sqrt{9+4+16} = \sqrt{29}$.

where $\vec{u} = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 4 \end{bmatrix} \in \mathbb{R}^4$

<u>unit vector:</u> length = 1 $\qquad \vec{u}_{unit} = \dfrac{\vec{u}}{\|\vec{u}\|}$

Product of two vectors? Dot product gives a scalar, cross product gives a new vector.

Dot product: $\vec{u} \cdot \vec{v}$

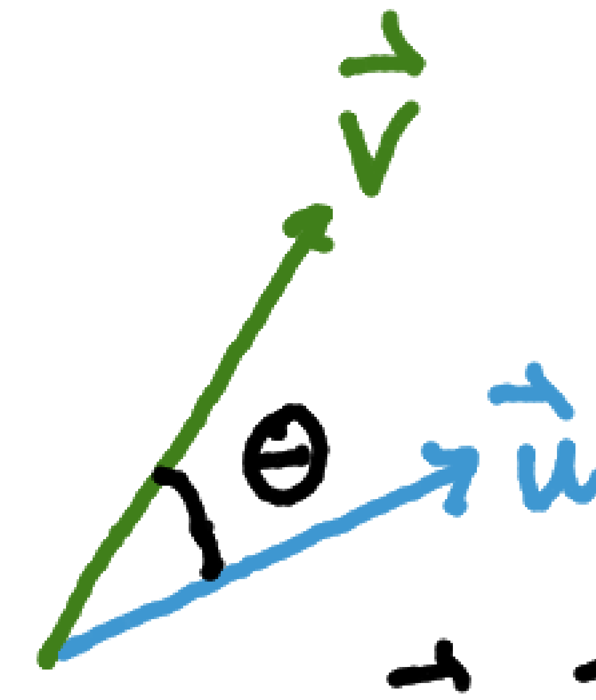
$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + \ldots$

$= \|\vec{u}\| \, \|\vec{v}\| \cos\theta$

(scalar)

if $\vec{u} \cdot \vec{v} = 0$? $\theta = 90°$

$\vec{u}, \vec{v}$ perpendicular (orthogonal)



Cross product: $\vec{u} \times \vec{v} = \vec{w}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \boxed{\vec{e}_x} & \boxed{\vec{e}_y} & \boxed{\vec{e}_z} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$$

$$= \vec{e}_x (u_y v_z - u_z v_y) - \vec{e}_y (u_x v_z - u_z v_x) + \vec{e}_z (u_x v_y - u_y v_x)$$

$=$ vector

$\vec{w}$ is perpendicular to $\vec{u}, \vec{v}$

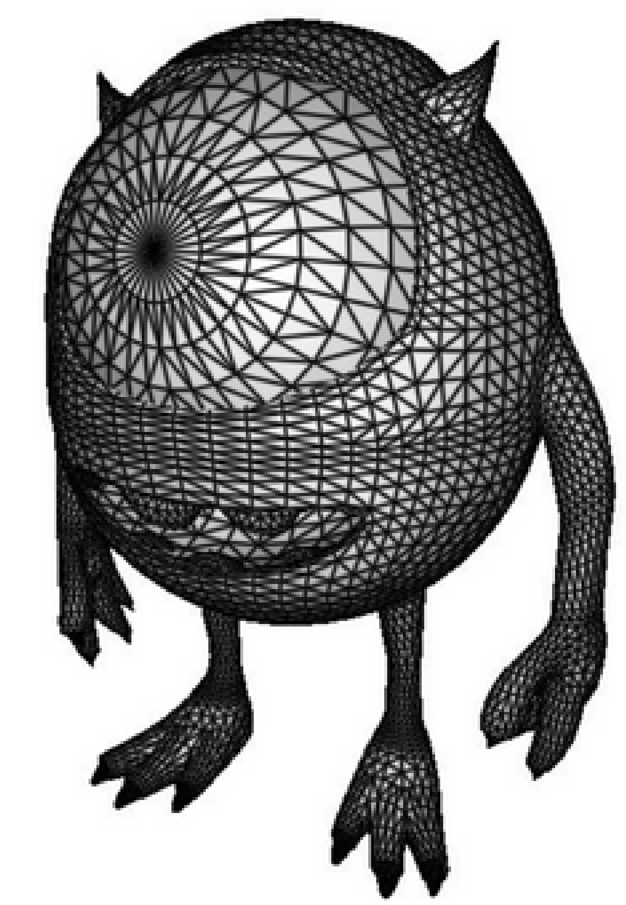
$\vec{w} \cdot \vec{u} = 0$

$\vec{w} \cdot \vec{v} = 0$

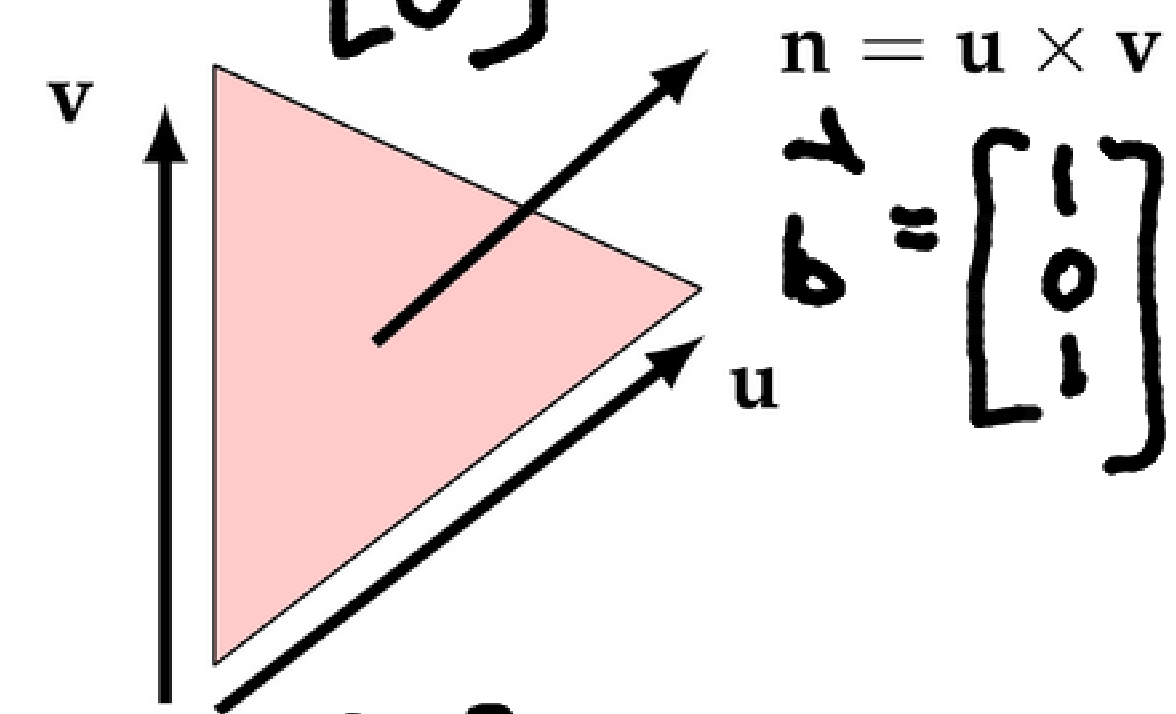
Example 1: calculate a unit vector perpendicular to this triangle.

① calculate $\vec{u} = \vec{b} - \vec{a} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\vec{v} = \vec{c} - \vec{a} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$



$\vec{c} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$



$\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\vec{a} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

② calculate $\vec{w} = \vec{u} \times \vec{v}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \hat{i}(0-1) - \hat{j}(0-1) + \hat{k}(1-0)$$

$$= (-1)\hat{i} + (1)\hat{j} + (1)\hat{k}$$

$$\vec{w} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\vec{w} \cdot \vec{u} = \underset{w_x}{(-1)}\underset{u_x}{(1)} + \underset{w_y}{(1)}\underset{u_y}{(0)} + \underset{w_z}{(1)}\underset{u_z}{(1)} = 0$$

What is the length of [redacted] u x v ? 22

- 1
- (3)^(1/3)
- 3^(1/2) $\sqrt{3} = (1+1+1)^{1/2}$
- 3

Voting as Anonymous

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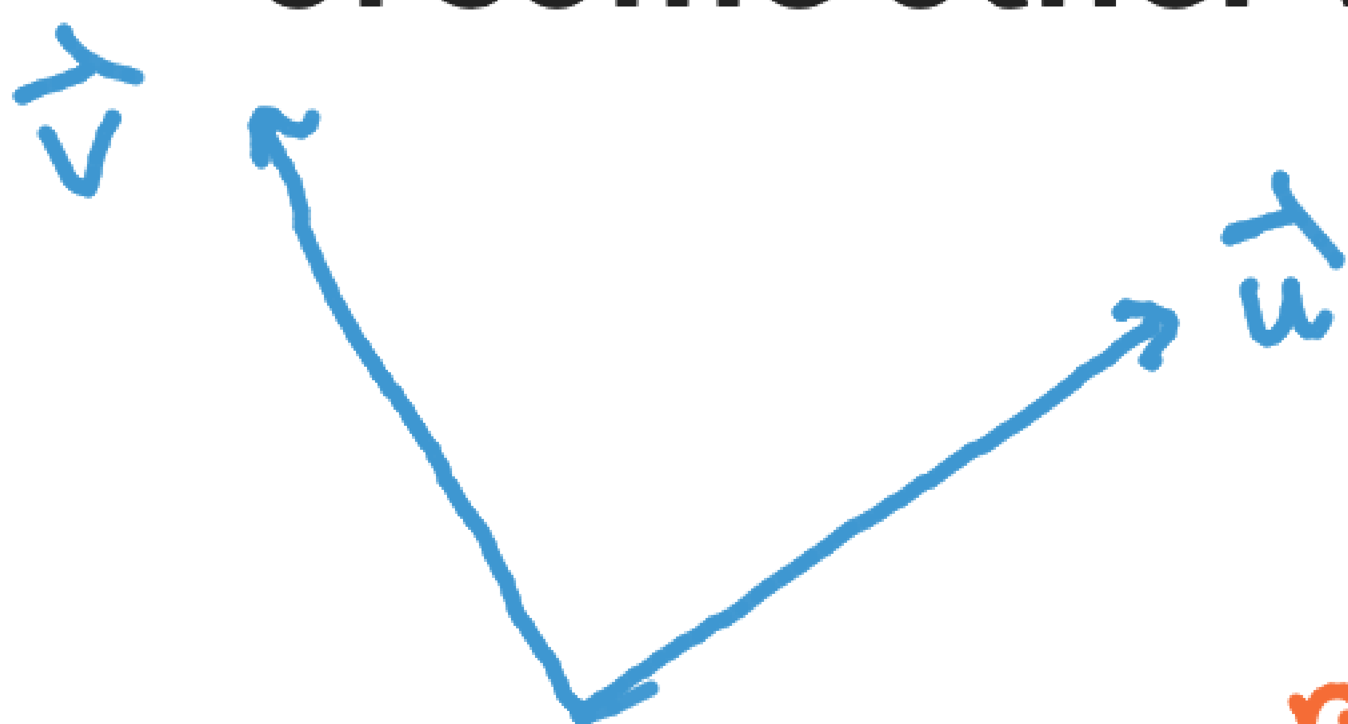
Transforming vectors: how much to "go" in direction of some other vectors (linear combination).



$\vec{w} = c_1\vec{u} + c_2\vec{v}$ $\quad c_1, c_2 \in \mathbb{R}$

represent as

$$\underbrace{\begin{bmatrix} \boxed{\vec{u}} & \boxed{\vec{v}} \end{bmatrix}}_{\text{matrix}} \underbrace{\begin{bmatrix} \boxed{c_1} \\ c_2 \end{bmatrix}}_{\text{vector}} = \vec{w}$$

matrix-vector multiplication

$$A\vec{c} = \vec{w}$$

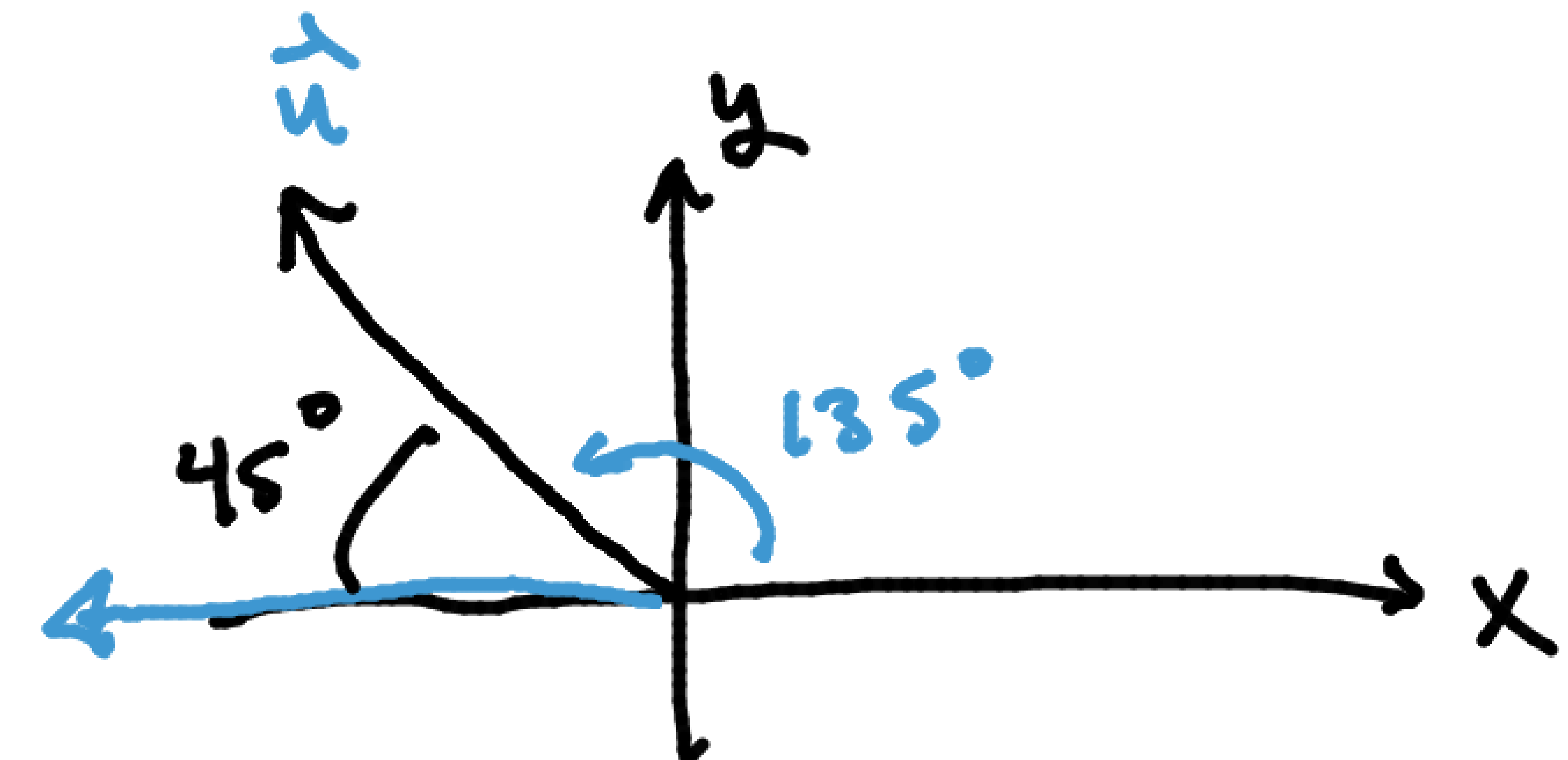
can "transform" vectors (scaling, rotate, reflect, project)

Example 2: what does this matrix-vector multiplication do? Use $\theta = 45^\circ$.

$\cos\theta = \frac{\sqrt{2}}{2}$

$\sin\theta = \frac{\sqrt{2}}{2}$

$$\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}\overset{\hat{u}}{\begin{bmatrix}-2 \\ 2\end{bmatrix}}$$



$$\begin{bmatrix}\boxed{\begin{matrix}\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2}\end{matrix}} & \boxed{\begin{matrix}-\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2}\end{matrix}}\end{bmatrix}\begin{bmatrix}\boxed{-2} \\ \boxed{2}\end{bmatrix} = \begin{bmatrix}-\sqrt{2} + -\sqrt{2} \\ -\sqrt{2} + \sqrt{2}\end{bmatrix} = \begin{bmatrix}-2\sqrt{2} \\ 0\end{bmatrix}$$

rotation matrix !