Try to form a conclusion from these statements:

If I go swimming, then I will stay in the sun too long. how??
If I stay in the sun too long, then I will get a sunburn.
I went swimming. Therefore, I will get a sunburn.

Can we structure this deduction?
How can we check our conclusion?

Goals for today

- identify **variables**, **premises** and **conclusions**,
- use a **truth table** to check your deductions,
- deduce new truths from existing statements using **rules of inference**.

If you already have a conclusion, check it with a truth table.

(a) *If I go swimming, then I will stay in the sun too long.*
(b) *If I stay in the sun too long, then I will get a sunburn.*
(c) *I went swimming. Therefore,* I will get a sunburn.

① label/identify variables: p, q, s

② identify premises:

(a) $p \to q$
(b) $q \to s$
(c) p

③ check column for your conclusion for all rows in which all premises are true.

truth table:

vars			premises			conclusion
p	q	s	$p \to q$	$q \to s$	p	s
T	T	T	T	T	T	T
T	T	F	T	F	T	
T	F	T	F	T	T	
T	F	F	F	T	T	
F	T	T	T	T	F	
F	T	F	T	F	F	
F	F	T	T	T	F	
F	F	F	T	T	F	

Four rules of inference to chain premises when forming new truths. (4 for us)

rule #1: vars: p, q

premises:

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$

"therefore" (conclusion)

vars		premises		
p	q	p	$p \to q$	q
T	T	T	T	T
T	F	T	F	
F	T	F	T	
F	F	F	T	

rule #2:

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

exercise

rule #3:

$$\begin{array}{l} \neg p \to \neg q \\ \hline \therefore q \to p \end{array}$$

rule #4:

$$\begin{array}{l} \neg q \\ p \to q \\ \hline \therefore \neg p \end{array}$$

Example: use rules of inference to form a conclusion.

If I go swimming, then I will stay in the sun too long. (a) p, q

If I stay in the sun too long, then I will get a sunburn. (b) q, s

I went swimming. Therefore, ________________. (c) p (true)

(a) $p \to q$ ✓
(b) $q \to s$ ✓
(c) p

step	statement	notes
1.	$p \to s$	rule #2 with (a), (b)
2.	s	rule #1 with (c) and step 1
	$\therefore s$	

Rule 1:	Rule 2:	Rule 3:	Rule 4:
p	$p \implies q$		$\neg q$
$p \implies q$	$q \implies r$	$\neg p \implies \neg q$	$p \implies q$
$\therefore q$	$\therefore p \implies r$	$\therefore q \implies p$	$\therefore \neg p$

Example: Will we be home by sunset?

① $\neg s \wedge c$ ✓
② $w \to s$ ✓
③ $\neg w \to p$ ✓
④ $p \to h$

It is not sunny this afternoon and it is colder than yesterday. We will go swimming only if it is sunny. If we do not go swimming, then we will take a canoe trip. If we take a canoe trip, then we will be home by sunset. Will we be home by sunset?

$w \to s$

- s: it is sunny
- c: it is colder than yesterday
- w: we will go swimming
- p: we will take a canoe trip
- h: we will be home by sunset

step	statement	notes
1.	$\neg s \wedge c$	P1
2.	$\neg s$	simplify step 1
3.	$\neg w$	step 2 with P2 + rule 4
4.	p	rule 1 with P3 + step 3
	$\therefore h$	rule 1 with P4 + step 4

Rule 1:

$$\begin{array}{c} \boxed{p} \; \neg w \\ p \implies q \\ \hline \therefore q \end{array}$$

Rule 2:

$$\begin{array}{c} p \implies q \\ q \implies r \\ \hline \therefore p \implies r \end{array}$$

Rule 3:

$$\begin{array}{c} \neg p \implies \neg q \\ \hline \therefore q \implies p \end{array}$$

Rule 4:

$$\begin{array}{c} \neg q \\ p \implies q \\ \hline \therefore \neg p \end{array}$$

IMPLICATIONS

We will discuss several other important ways in which propositions can be combined.

Let p and q be propositions. The *implication* $p \rightarrow q$ is the proposition that is false when p is true and q is false, and true otherwise. In this implication p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

The truth table for the implication $p \rightarrow q$ is shown in Table 5. An implication is sometimes called a **conditional statement.**

Because implications play such an essential role in mathematical reasoning, a variety of terminology is used to express $p \rightarrow q$. You will encounter most if not all of the following ways to express this implication:

"if p, then q"	"p implies q"
"if p, q"	"p only if q"
"p is sufficient for q"	"a sufficient condition for q is p"
"q if p"	"q whenever p"
"q when p"	"q is necessary for p"
"a necessary condition for p is q"	"q follows from p"

The implication $p \rightarrow q$ is false only in the case that p is true, but q is false. It is true when both p and q are true, and when p is false (no matter what truth value q has).

A useful way to understand the truth value of an implication is to think of an obligation or a contract. For example, the pledge many politicians make when running for office is: