Union of three sets (example from last class):

Track the curved triangle!

\[ |S| + |C| + |G| - |S \cap G| - |S \cap C| - |C \cap G| + |S \cap C \cap G| \]
Quantifiers can be used to quantify the values of predicates.

\[ \forall x \in \mathbb{N}, \ x > 0 \]

\( \forall \) means "for all"

\[ \exists x \in \mathbb{N} : x < 0 \]

\( \exists \) means "exists"

"such that"
Example 1: translate Some birds can fly.

Let $A$ be the set of all animals.
Let $b(x)$ be the predicate that $x$ is a bird.
Let $f(x)$ be the predicate that $x$ can fly.

\[ \exists x \in A : (b(x) \land f(x)) \]

is this okay? \[ \exists x \in A : b(x) \Rightarrow f(x) \]

$x$ is an elephant

\[ b(x) \quad F \quad F \Rightarrow F \quad \text{is T} \]

\[ f(x) \quad F \]
Negating quantified expressions.

Every Middlebury student lives on campus.

\[ \forall x \in S, \ p(x) \]

\( p(x) \) : \( x \) lives on campus

\( S \) : students

what about:

Not every student lives on campus.

\[ \neg (\forall x \in S, \ p(x)) \]

There exists one student who does not live on campus.

\[ \exists x \in S : \neg p(x) \]

\( \neg (\forall x \in S, \ p(x)) \equiv \exists x \in S : \neg p(x) \)

\( \neg (\exists x \in S : p(x)) \equiv \forall x \in S, \neg p(x) \)
Some tips!

- Combine multiple elements of same type with a quantifier:
- Any non-quantifier variables should be inputs to a predicate,
- When a variable is quantified, rewrite your predicate in terms of remaining variables: \( \exists y \in S: p(x,y) \) rewrite as \( q(x) \)
- All variables in a statement should be quantified.
Example 2: Determine whether the following statements are true or false.

Let $S$ be the set of all people. Let $p(x, y)$ be the predicate that $x$ is a parent of $y$.

- $\forall x \in S, \exists y \in S: p(x, y)$ every person has a child $\quad \text{F}$
- $\forall x \in S, \exists y \in S: p(y, x)$ every person has a parent $\quad \text{T}$
- $\exists x \in S, \forall y \in S: p(y, x)$ one person is the child of all people $\quad \text{F}$
Example 3: Translate the following to math.

Let $S$ be the set of people in the class.
Let $F(x, y)$ mean that person $x$ considers person $y$ to be their friend ($x \neq y$).

1. Proposition $p$ states that there is some super likable person in the class that everyone considers their friend.
   \[ \exists x \in S \forall y \in S, \ F(y, x) \]

2. Proposition $q$ states that everyone in the class has at least one person they consider to be their friend.
   \[ \forall x \in S \exists y \in S \ F(x, y) \]

3. Proposition $r$ states that there is a mutual friendship in the class.
   \[ \exists x, y \in S : \ F(x, y) \land F(y, x) \]

4. Predicate $(b)$ states that everyone who considers person $x$ to be their friend also considers person $y$ to be their friend.
   \[ \forall z \in S, \ F(z, x) \rightarrow F(z, y) \]