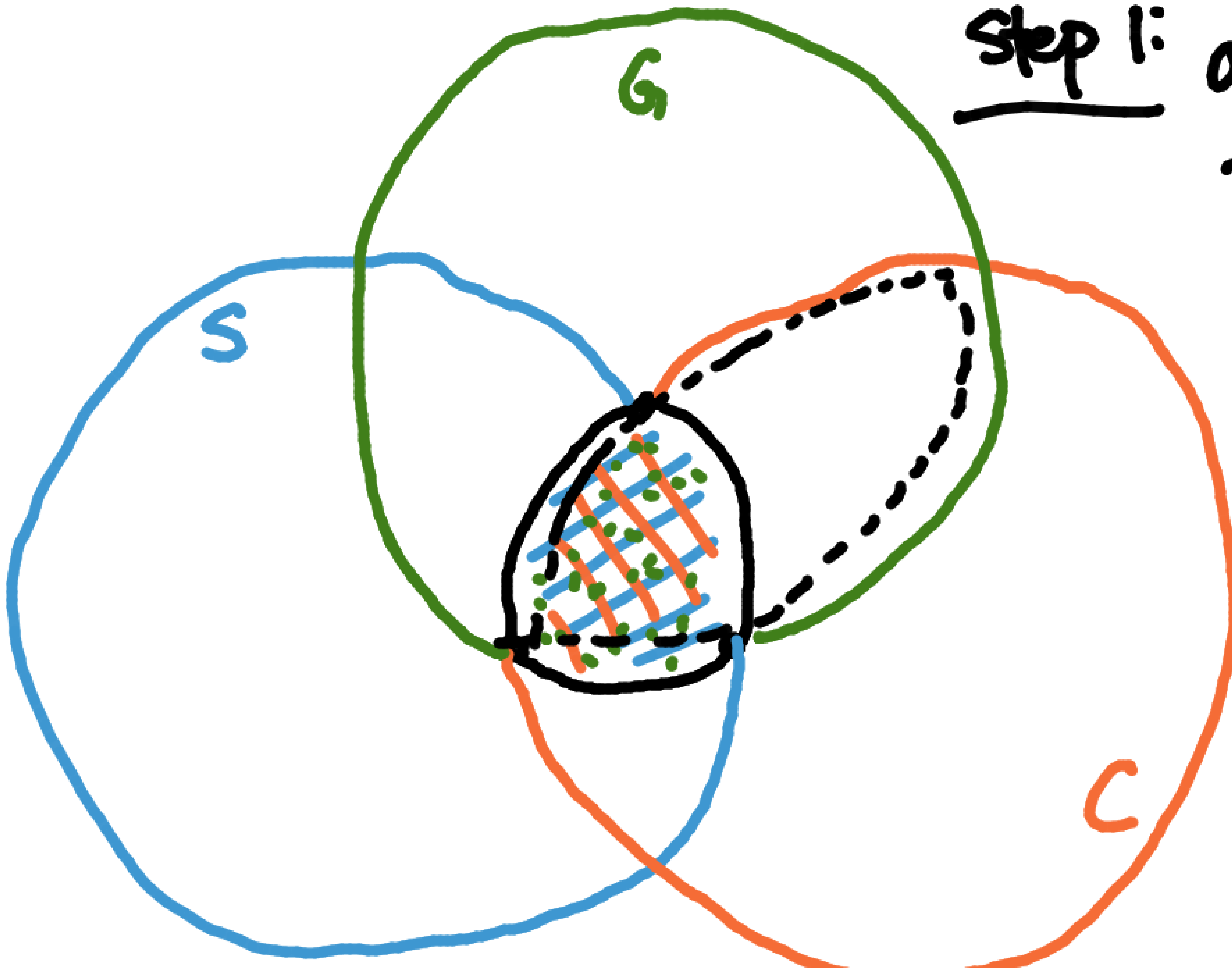
Union of three sets (example from last class):

track the curved triangle!



Step 1: add contribution from the cardinality of each set

Step 2: subtract contribution from

$|S| + |C| + |G| - |S \cap G| - |S \cap C| - |C \cap G|$
$+ |S \cap C \cap G|$

Quantifiers can be used to quantify the values of predicates.

"for all"

universal quantifier

domain

$$\underbrace{\forall x \in \mathbb{N},\ \underbrace{x > 0}_{\text{predicate}}}_{\text{statement (True)}}$$

$\forall$ means "for all"

\forall

"exists"

existential quantifier

domain

"such that"

$$\underbrace{\exists x \in \mathbb{N} :\ \underbrace{x < 0}_{\text{predicate}}}_{\text{statement (False)}}$$

$\exists$ means "exists"

\exists

Example 1: translate *Some birds can fly.*

Let A be the set of all animals.
Let $b(x)$ be the predicate that x is a bird.
Let $f(x)$ be the predicate that x can fly.

<u>ex:</u> want this to be true.

$$\exists x \in A : (b(x) \wedge f(x))$$

<u>is this okay:</u> ? $\exists x \in A : b(x) \to f(x)$

x is an elephant

$b(x)$ F

$f(x)$ F

F → F is T

Negating quantified expressions.

S: students

Every Middlebury student lives on campus.

p(x): x lives on campus

$$\forall x \in S,\ p(x)$$

what about:

Not every student lives on campus.

$$\neg(\forall x \in S,\ p(x))$$

There exists one student who does not live on campus

$$\exists x \in S : \neg p(x)$$

$$\neg(\forall x \in S, p(x)) \equiv \exists x \in S : \neg p(x)$$

$$\neg(\exists x \in S : p(x)) \equiv \forall x \in S, \neg p(x)$$

Some tips!

$\forall x, y \in S, p(x,y)$

- Combine multiple elements of same type with a quantifier:
- Any non-quantifier variables should be inputs to a predicate,
- When a variable is quantified, rewrite your predicate in terms of remaining variables: $\exists y \in S: p(x,y)$ rewrite as $q(x)$
- All variables in a statement should be quantified.

Example 2: Determine whether the following statements are true or false.

Let S be the set of all people. Let $p(x, y)$ be the predicate that x is a parent of y.

- $\forall x \in S, \exists y \in S\colon\ p(x, y)$ every person has a child F
- $\forall x \in S, \exists y \in S\colon\ p(y, x)$ every person has a parent T
- $\exists x \in S, \forall y \in S\colon\ p(y, x)$ one person is the child of all people F

Example 3: Translate the following to math.

Let S be the set of people in the class.
Let $F(x, y)$ mean that person x considers person y to be their friend ($x \neq y$).

1. Proposition p states that there is some super likable person in the class that everyone considers their friend.

$$\exists x \in S \; \forall y \in S, \; F(y, x)$$

2. Proposition q states that everyone in the class has at least one person they consider to be their friend.

$$\forall x \in S \; \exists y \in S \quad F(x, y)$$

3. Proposition r states that there is a mutual friendship in the class.

$$\exists x, y \in S : F(x,y) \wedge F(y,x)$$

4. Predicate $b(x,y)$ $\cancel{b(x)}$ states that everyone who considers person x to be their friend also considers person y to be their friend. $\forall z \in S, \; F(z,x) \rightarrow F(z,y)$