Logical operators: $\land$, $\lor$, $\neg$

Let $p = I$ am wearing a blue shirt today.
Let $q = Today$ is Monday.

$p \land q = I$ am wearing a blue shirt AND today is Monday.

$p \lor q = I$ am wearing a blue shirt OR today is Monday.

$\neg p = I$ am NOT wearing a blue shirt.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \land q$</th>
<th>$p \lor q$</th>
<th>$\neg p$</th>
</tr>
</thead>
<tbody>
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Implications: think of a "promise".

Let $p = \text{You get 100% on the final.}$
Let $q = \text{You get an A in the course.}$

"if $p$, then $q$" $p \implies q$

is this promise valid?

```
in LaTeX
\implies

if-and-only-if: biconditional

\iff p \iff q
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$p \iff q = (p \implies q) \land (q \implies p)$

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P \quad q \quad p \implies q \quad q \implies p \quad p \iff q
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$p \iff q$ is false only time $p \implies q$ is false.
Proving with a truth table.

- Identify and label smaller propositions (with variables).
- Write overall proposition symbolically.
- Make a column for each variable and build up to overall proposition.

**Example:** Prove the following: *If you eat spinach everyday, then you will win the lottery, or if you win the lottery, you will lose your job.*

\[
(p \rightarrow q) \lor (q \rightarrow r)
\]

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<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>p \rightarrow q</th>
<th>q \rightarrow r</th>
<th>(p \rightarrow q) \lor (q \rightarrow r)</th>
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Every row is true.
de Morgan's laws: a tool to simplify compound propositions.

\neg (p \land q) \equiv \neg p \lor \neg q

\neg (p \lor q) \equiv \neg p \land \neg q

how to prove? build truth table.