Logical operators: $\wedge$, $\vee$, $\neg$

Let p = *I am wearing a blue shirt today.*
Let q = *Today is Monday.*

$p \wedge q$ = I am wearing a blue shirt AND today is Monday.
↑ AND ("∧" looks like A in AND)

$p \vee q$ = I am wearing a blue shirt OR today is Monday
↑ OR

$\neg p$ = I am NOT wearing a blue shirt.

truth table

p	q	$p \wedge q$	$p \vee q$	$\neg p$
T	T	T	T	F
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Implications: think of a "promise".

Let p = *You get 100% on the final.*
Let q = *You get an A in the course.*

"if p, then q" $p \to q$

is this promise valid?

in LaTeX
\implies

p	q	p → q
T	T	T
T	F	F ← only time p→q is false.
F	T	T
F	F	T

if-and-only-if: biconditional $p \leftrightarrow q$

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

p	q	p→q	q→p	p↔q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Proving with a truth table.

$(p \to q) \vee (q \to r)$

- Identify and label smaller propositions (with variables).
- Write overall proposition symbolically.
- Make a column for each variable and build up to overall proposition.

Example: Prove the following: *If you eat spinach everyday, then you will win the lottery or if you win the lottery, you will lose your job.*

p	q	r	p → q	q → r	(p → q) ∨ (q → r)
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

every row is true.

de Morgan's laws: a tool to simplify compound propositions.

distribute ¬ and flip ∧ to ∨

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

how to prove? build truth table.