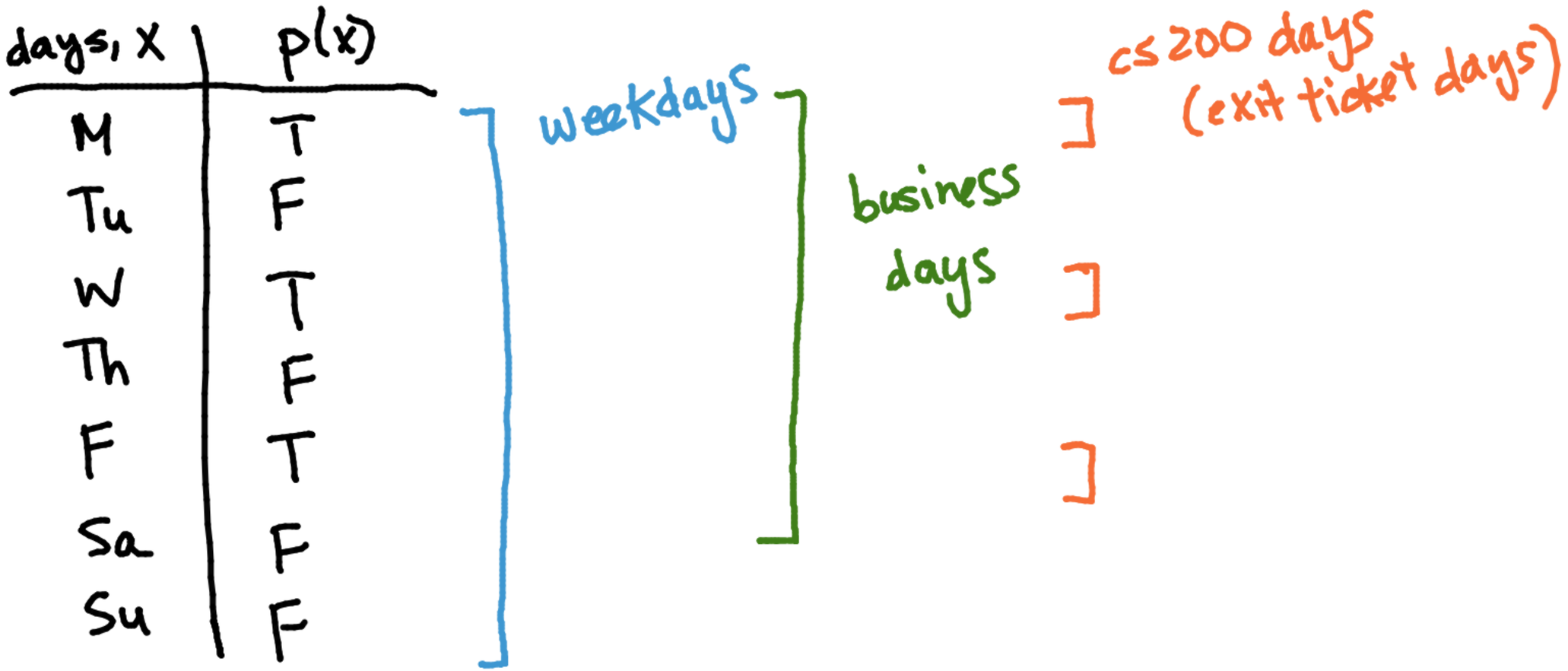
Last time: predicates (propositions that depend on variables).

Example: $p(x) =$ *Exit tickets are due on* x.

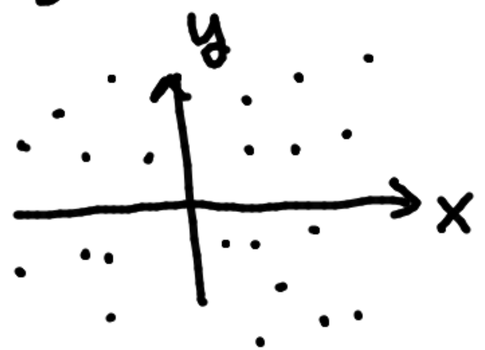


A *set* is a bunch of objects, called *elements*.

- set of animals: $\{$ cat, dog, penguin, elephant, ... $\}$

\lbrace \rbrace

- set of even numbers: $\{2, 4, 6, 8, 10, \ldots\}$



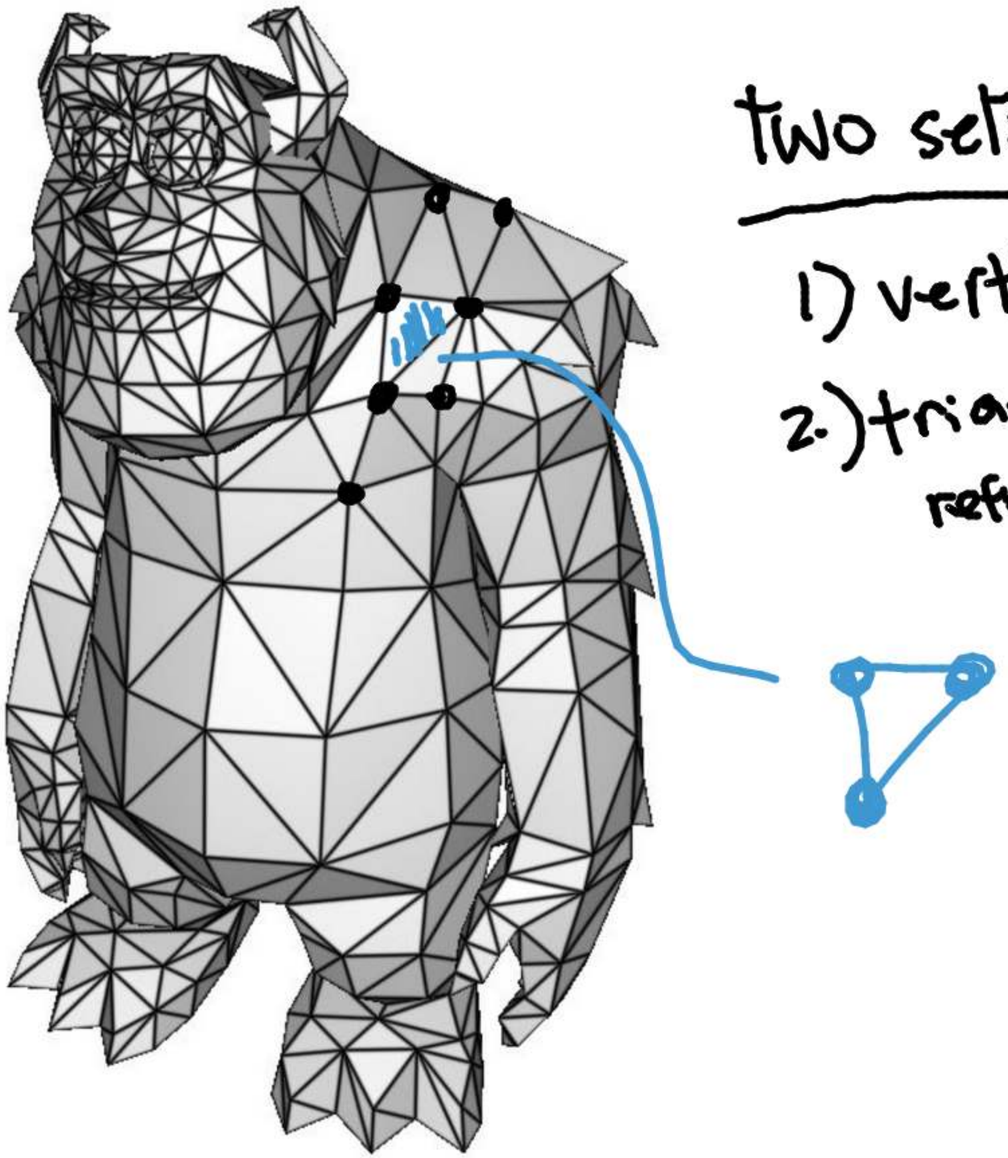
- set of points on the plane: $\{(0,0), (1,0), (1,1), \ldots\}$

- set of integers: $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$ $\mathbb{Z}$
- set of natural numbers: $\{1, 2, 3, \ldots\}$ $\mathbb{N}$ some people include 0, we won't
- set of real numbers: $\{1.1, 3.1415, \sqrt{2}, e, \ldots\}$ $\mathbb{R}$

number of elements in a set A is called the cardinality : $|A|$

\lvert \rvert

Set of triangles representing a character!



Describing sets: roster notation.

↳ explicitly listing all elements in set

weekdays: $\{M, Tu, W, Th, F, Sa, Su\}$

positive even numbers up to 10: $E = \{2, 4, 6, 8, 10\}$

order doesn't matter
all elements unique

element is in a set: $\in$ $\quad 2 \in E \quad 7 \notin E$

↑ \in ↑ \notin

Describing sets: set-builder notation (predicates!).

↳ use a predicate to filter elements from some domain

$$A = \{ x \in \mathbb{Z} \mid x > 0 \}$$

domain; "such that"; predicate

→ sometimes we'll use :

$$B = \{ x^2 + 2 \mid x \in \mathbb{R} \}$$

$$\mathbb{Q} = \left\{ \frac{x}{y} \mid x, y \in \mathbb{Z} \wedge y \neq 0 \right\}$$

↑ rational numbers

Practice: let $A = \{x^2 \mid x \text{ is even}\}$.

Other ways to express A (mathematically)?

$$A = \{ x^2 \mid x \% 2 == 0 \}$$

$$A = \{ x^2 \mid \frac{x}{2} \in \mathbb{Z} \}$$

$$A = \{ (2n)^2 \mid n \in \mathbb{Z} \}$$

	even	odd
n	2n	2n+1
1	2	3
2	4	5
3	6	7
4	8	9
5	10	11
6	12	13
⋮	⋮	⋮

Subsets and operations.

let A, B be sets

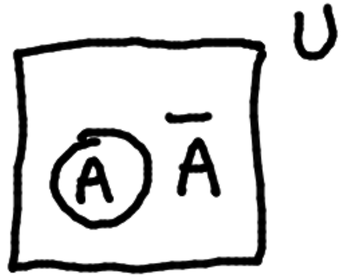
$A \subset B$ (strict)

$A \subseteq B$

e.g. U = integers, $\mathbb{Z}$

B = positive integers $\mathbb{Z}^+$

A = positive even integers

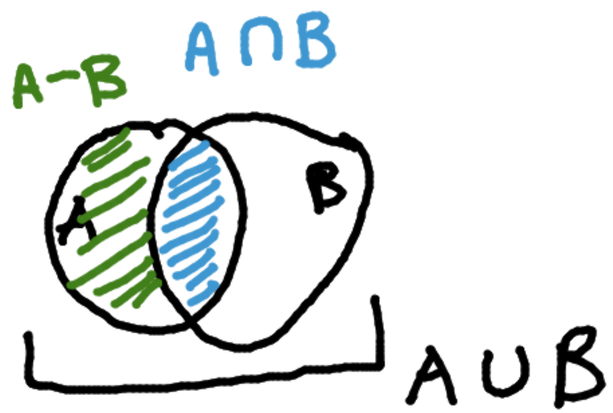


complement: $\bar{A} = U - A = \{x \in U \mid x \notin A\}$

union: $A \cup B = \{x \mid x \in A \vee x \in B\}$

intersection: $A \cap B = \{x \mid x \in A \wedge x \in B\}$

difference: $A - B = \{x \mid x \in A \wedge x \notin B\}$



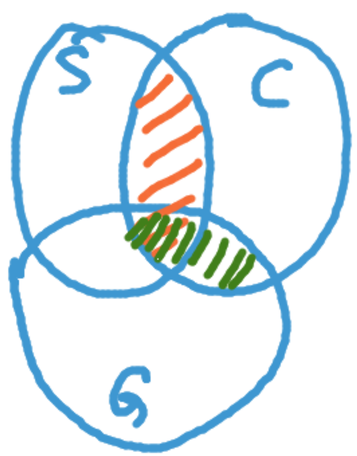
Revisiting our first question:

You decide to poll everyone in the class for what kind of candy they like. There are three options: sour candy (S), chocolate (C) and gummies (G). Everyone can enter multiple choices, including: S, C, G, SC, SG, CG or SCG.

Results of the poll:

S	C	G	SC	SG	CG	SCG
0	0	1	6	3	1	20

How many candies should you buy?



$$|S| + |C| + |G| - |S \cap C| - |C \cap G| - |G \cap S| + |S \cap C \cap G|$$